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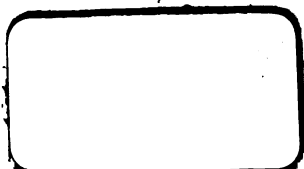
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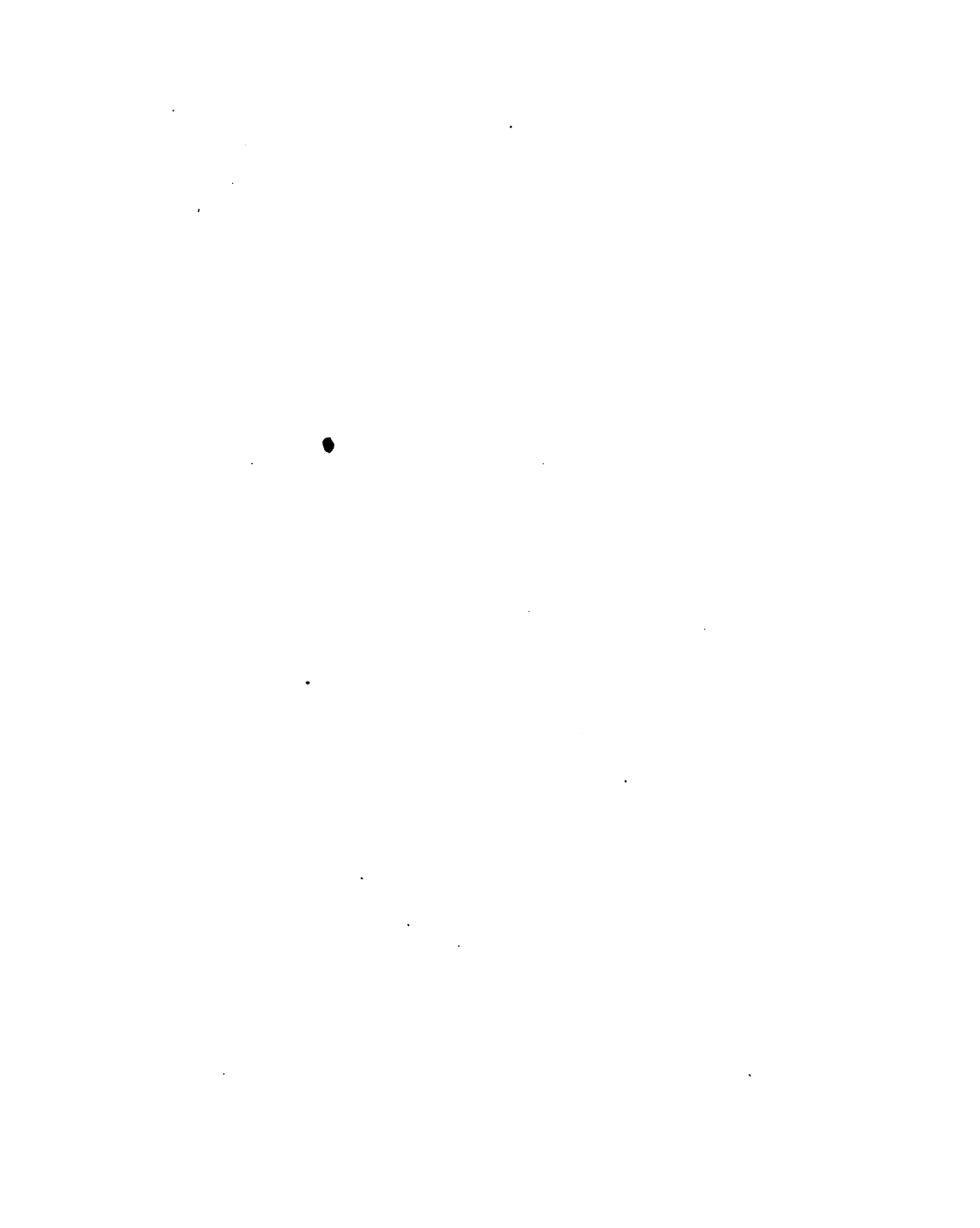


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NAUTICAL SURVEYING

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AN INTRODUCTION TO THE
PRACTICAL AND THEORETICAL STUDY
OF
NAUTICAL SURVEYING

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P R E F A C E.

ONE of our most experienced and skilful surveyors has laid down the *dictum* that no officer can be fully qualified to take charge of the soundings of a survey with less than seven years' practice. This opinion is perhaps somewhat exaggerated, though less so than it will at first sight seem to many. In all surveying operations, practice and experience can alone give skill ; nevertheless, it is not to be doubted, that without a sound knowledge of the theoretical principles on which the various operations are based, the teachings of practice and experience are given to very little purpose ; and that without proper guidance, much practice may be thrown away on objects utterly useless.

In this Introduction to the Study of Nautical Surveying I have endeavoured to impart such a knowledge of principles, and to offer such guidance

to practice, as will enable the young surveyor more readily to arrive at the end which he has in view—a mastery of the science ; and as will enable others to appreciate more exactly and intelligently the result of the surveyor's work. It is, however, only as an *Introduction* that I offer it ; and many problems of great interest and value, as well as the discussion of many refinements of correction and reduction, which, neat as they may be in theory, are of very little practical use, have been necessarily and purposely omitted. I have thus also laid what may perhaps be considered undue stress on observations with the sextant, to the comparative exclusion of the theodolite. The sextant is, in fact, more likely to be in the hands of a young officer : it is an instrument of which he ought to have a perfect mastery ; all the astronomical observations of a survey, all the determinations of positions afloat must be made with it ; the observations on shore, connected with the triangulation and the determination of secondary stations, may be made with it ; in short, a complete survey may be carried on with the sextant alone, but not with the theodolite : and, besides, any one who can really use the sextant, will learn the ready and

familiar use of the theodolite in a couple of days' practice. A very experienced surveying officer has given it to me as his opinion on this subject, that a young surveyor ought not to be allowed to touch the theodolite till he has proved himself capable of doing all the work, in a satisfactory manner, with the sextant : and there is little doubt that if he does not attain a perfect use of the sextant, for such observations, before he begins to use the theodolite, he is not likely to do so after.

It is scarcely necessary to say that a work of this kind has small pretensions to originality. Much of the matter has been gathered out of various published treatises on the same or kindred subjects ; some has been worked out simply on mathematical principles ; most, perhaps, has been picked up in the course of conversation with friends of the surveying branch of the Service. I can only acknowledge the kindly assistance I have received from the officers of the Hydrographic Department of the Admiralty, and, still more, from my fellow-worker here, in the Royal Naval College, Staff-Commander V. F. Johnson.

J. K. LAUGHTON.

PORTSMOUTH : *June 17, 1872.*



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NAUTICAL SURVEYING.



CHAPTER I.

INTRODUCTION.

Definition of Surveying—Specialities of a Nautical Survey—
Its peculiar difficulties—Scope of the present work.

SURVEYING is the method by which we acquire such a knowledge of a country, or any part of the earth's surface, as to be able to lay it down on a map or chart.

According to the nature of the survey, and the purposes for which it is made, maps or charts may have many degrees of accuracy. An experienced observer and good draughtsman can make such a survey of a country, in a hasty ride through it, and a view from some of the principal heights, as to be able to draw it in a free manner, which will give a good general idea of it, sufficient in many instances for military purposes, or the wants of a traveller. But when a correct and detailed knowledge of bearings and distances is required, such as is necessary for permanent maps of any country, or

a

for charts on which the safety of our ships continually depends, this has to be obtained by observers using exact instruments, so as to measure all the required data with the greatest possible accuracy ; and it is the conduct of observations of this class, their correction, reduction, and proper grouping, which constitutes what is now strictly understood as SURVEYING.

In its closest sense, the theory of surveying is merely a branch of Trigonometry. When two positions are thoroughly determined, the distance between them being exactly known can be considered as the base of a triangle, the vertex of which is any other visible point ; and if the angles formed by the base and lines drawn to this point be observed, the triangle may either be solved by the ordinary formulæ, or be laid down at once on paper, by means of the protractor. The sides of this triangle, being thus known, may be taken for new bases ; and a system of triangles may be extended over any country, however large.

In the theory of such work there is no difficulty whatever ; it is throughout simply the solution of triangles and the determination of angles and distances. The difficulty of surveying lies wholly in the practice of it. Observations are always more or less faulty ; the imperfections of instruments and of eyesight do not permit them to be otherwise ; and the detection, the reduction, or the elimination of errors offers a wide field for the ingenious theorist.

These errors, again, are theoretically shown to have greatest or least values, or to exercise greater or less

influence on the result under different calculated circumstances, a knowledge of which is most important, tending, as it does, to promote the final accuracy of the work.

Theory and practice here go hand in hand ; and the selection of the different points to be observed, the comparison of the advantages or disadvantages offered by different methods of observing, accuracy in observing and in noting down the observations so that they may be readily interpreted by the draughtsman, altogether require a degree of skill which only practice and a naturally quick eye can give.

The conduct of a nautical survey differs from that of a land survey principally in this, that as triangulation frequently cannot be carried on from one part of a coast to another, or to an outlying island, it is continually necessary to make a fresh start, and to determine positions independently ; whilst on shore, when a beginning has once been made, the network of triangles may be extended over the whole face of the country. More, therefore, depends on strict accuracy in determining the initial positions for a land survey, and necessitates the use of the most exact instruments, whose large size and delicate adjustments render it impossible either readily to convey them from place to place, or to set them up wherever occasion may require. But in the survey of a line of coast for nautical purposes, whether from the imperfections of the instruments, or from the hurried manner in which the observations have frequently to be made, anything like perfect accuracy in the absolute position of stations

is not attainable, nor indeed is it necessary ; what is principally required is accuracy in the relative position of the points laid down—accuracy in their bearings and distances one from the other ; and as long as this accuracy is obtained within really practical limits, it is as much as we can expect with the instruments, whether sextant or portable theodolite, which we are compelled to use.

And the work of a nautical survey is carried on, not only with instruments which from their very nature do not admit of minute accuracy, but most commonly also as a race against time ; and where the observations are hurried, it is impossible to eliminate the errors which necessarily creep in, by taking the mean of a very great number ; it is impossible to wait for weather, so that observations known to be inferior have often to be used, simply because the proper observations cannot be taken.

In the great trigonometrical surveys, on the other hand, the exact observations required are taken at any cost of time ; as one instance of which I may mention that on the summit of Sca-fell Pikes, the highest peak in England, the party of engineers employed on the ordnance survey remained for about six weeks, waiting to complete their observations, and especially to connect the triangulation of Ireland with that of England. In a nautical survey, such a delay would be out of the question, and the connection would have to be made by observations of a less rigorous nature.

And yet the most perfect accuracy is desirable. In this lies a great part of the peculiar difficulty of a nautical

survey. With imperfect instruments, scant allowance of time, and in weather that may or may not be favourable, to carry out a survey with the very close accuracy which is now insisted on, and which most of our recent surveys possess, is a work that requires not only an extraordinary amount of skill in every branch of the art, practical as well as theoretical, but also physical and personal qualities of a high order—quickness of eyesight, steadiness of nerve, and evenness of temper ; without which it is impossible for a man to become a first-rate surveyor. These qualities are, however, not known till they are fairly tested ; skill and judgment in observing are not obtained without long and careful practice ; and with this, most men may attain a pitch of respectable proficiency, even though they may be incapable of attaining peculiar excellence.

But few, or perhaps none, who do not propose to attach themselves specially to the hydrographic department of the service have any opportunity of this practice, or of attaining any high degree of proficiency. Nor, indeed, is it requisite ; but a certain limited acquaintance with both the practice and theory of surveying is a necessary part of the training of every naval officer, without which he cannot have an intelligent understanding of the charts, the methods of using them, and the confidence to be placed in them.

This much may be learned without difficulty ; but I believe that we have long suffered from the want of an elementary work on the subject—of a work written not for the guidance of surveyors, but for the instruction of

those who know little or nothing at all about it. This want I have endeavoured to supply. If I have done so in a manner that will effectively aid those who wish to attain merely the very limited amount of knowledge of which I have just spoken, and will, at the same time, constitute a satisfactory groundwork for those who wish to advance further into the study of the science, I shall congratulate myself on having done all that I meant to do.

CHAPTER II.

OBSERVATIONS ON SHORE.

Choice of base line—Its determination—The artificial horizon—
 The sextant—Determination of latitude—Longitude—Meridian
 distances—Measurement of base—Length of base by sound—
 Determination of true bearings—Choice of primary stations—
 The triangulation—Secondary stations—Outline of coast—Deter-
 mination of heights—Levelling.

IN commencing the survey of any coast line, and more especially a coast of limited extent, as, for instance, of a bay, of a confined harbour, or of a small island, it is always advisable, so far as possible, to obtain, in the first instance, a good general idea of the coast to be surveyed. This may be done by riding or walking over the ground, by ascending such eminences as command a good view, or by pulling round the coast in a small boat. A roughly sketched map drawn on the summit of a hill and corrected by actual knowledge of the lower country, is of the greatest possible assistance in the preliminary work which has to be gone through, more especially in directing the choice of stations—that is, of positions at which exact observations are to be made, whether with theodolite or sextant.

Of these stations, the most important, and those requiring the most careful choice, are the first two, which, as already mentioned, are to form the foundation of all future calculations or protraction. As the distance between these two, when once chosen and properly determined, is used as the *base* of the first triangles afterwards selected and laid off, it is called the **BASE LINE** of the survey. The whole survey is, in fact, referred to it; and the accuracy of the work very much depends on the proper choice of the stations which form its extremities, and on the exactness with which their position, both *absolute* and more especially *relative*, is determined. The most essential points to consider in selecting these stations are,

1st. That the ground between them be smooth and level.

A level plain, such as is often found near the shore more especially at the mouth of a river; a marsh, if reasonably dry; the line of the shore itself, above high-water mark, may, as opportunity offers, require to be chosen; but the immediate neighbourhood of the sea, such as the shore line, is not always to be recommended; for gales of wind and driving spray may make it difficult to observe from either of the stations, and may render it impossible to establish any observatory near them, even if time and appliances should permit. On the other hand,

2nd. The stations should not be far from the shore. If at any distance, they are likely to be at a height, more or less considerable, above the sea level; a condition that is to be avoided, as it necessitates the reduction of many of

the observations, and thus causes a certain increase of work.

3rd. It is well that a fair extent of the coast, and the country to be surveyed, should be visible from each of the two stations, so that, especially in the beginning of the work, observations may be repeated from many different stations, and thus afford a check on each other, and a ready means of detecting any error.

4th. It is convenient to choose the stations so that the base line may run straight on to some distant and well-defined object ; such a direction renders it easier to find, or, as it is called, to 'pick up,' the stations from a distance, their bearing from some prominent and unmistakable object being known.

It is of course frequently impossible to find stations which fulfil all the conditions just mentioned ; but they should all be borne in mind and sought after, so that the relative advantages or disadvantages of different lines which suggest themselves may be properly taken into consideration.

The length of the base line may vary according to circumstances ; for a survey of very limited extent, about half a mile or even less might be sufficient ; for larger surveys, a greater length should be obtained ; but two miles is perhaps long enough for any detailed work likely to fall to the lot of the nautical surveyor. The difficulty of measuring the line exactly increases very much as its length increases, so that it is necessary to keep it within strictly reasonable limits ; indeed, many experienced surveyors recommend that the line measured

should not much exceed half a mile ; and that if a longer base is requisite, its length should be determined by observation and protraction, in the way which will presently be described.

The stations are marked by a pole, with a flag of *noticeable* colour, such as white, yellow, or red ; a dark colour, such as blue, is, at a little distance, lost in the background, and cannot be readily ‘picked up.’ A white-washed hamper is sometimes used instead of a flag ; it is perhaps easier to see, if the wind happens to be blowing straight from the observer to the station ; on the other hand, the flickering of the flag makes it generally more noticeable than the perfectly still basket. But the choice between the two is principally a matter of personal preference.

The poles with their distinguishing flags being set up at the stations selected, the work of determining their position has to be carried out. The full determination of the base line involves,

1. The latitude and longitude of *one* extremity.
2. The length of the line.
3. Its direction, that is, its line of bearing.

It may seem as if the whole determination might be more simply made by determining the latitude and longitude of *each* position ; and theoretically, indeed, this would be the case. But, practically, this is impossible, except in very large surveys, which aim merely at laying down the general features of the coast, without pretending to any exactness of detail. Surveys in which the base line may be so determined will be spoken of again, but for

any detailed work the method is useless ; and for this reason ; that the best sextants are only cut to 10'', and observations for latitude and longitude cannot be depended on as correct within that limit, even supposing that sextant, chronometer, and observer are all faultless ; and though by much care and trouble, the error arising from this source may be eliminated to a very great extent, it can never be certain that it is so entirely ; and the residual error at each station may be sufficient to alter the *relative* position very materially, though the *absolute* position of each is determined with sufficient accuracy. An error of 10'' in latitude would be equivalent to 1,000 feet ; a similar error in longitude might amount to as much, and would seldom be less than 500. It is quite clear that if such errors, trifling as they are in the observation, were to occur at each of the stations, and in opposite directions, the length of the base line so determined would be wrong by 2,000 or 3,000 feet, and would, of course, be utterly useless for any purpose at all. Besides this, the labour of determining the latitude and longitude of a second station by astronomical observations, is very much greater than that of referring it to the first, by careful measurement of its distance and bearing.

But on the correctness of the latitude and longitude of the first position, the absolute correctness of the whole survey depends ; and from the first selection of the station, the observations should be commenced, and should be continued so long as opportunity permits. They should be taken in the immediate neighbourhood of the station pole, and, when possible, a large stone

or boulder firmly bedded in the earth should form the support of the artificial horizon ; it is to a great extent free from the tremor which soft earth or living rock communicates to the mercury, and, being immovable, is a constant and permanent record of the exact place of observation.

The mercury used ought to be the purest obtainable ; if quite pure, it does not tarnish, at ordinary temperatures, by exposure to air and moisture, but if it contains other metals, the amalgam of those metals oxidises and collects as a film on the surface. Practically, the mercury never is quite pure, and such a film always does form ; it, as well as any dust, &c., which may have got in, may be removed by gently straining the mercury through a piece of chamois or other soft leather. Such a piece of leather may be kept in the horizon box, but it must never, on any account, be allowed to touch the sextant, for any mercury coming on the arc of a sextant would destroy the graduation and utterly ruin the instrument.

If, however, any quantity of foreign metal, such as tin or lead, has got in amongst the mercury, as may sometimes happen, either by accident or carelessness, after resilvering the glasses of the sextant, it turns the whole body of the mercury into an amalgam, which, as an artificial horizon, is quite useless, and which cannot be rendered bright by any amount of straining. It is a bad case, the only perfect remedy being to distil the mercury, a process which needs great care, proper apparatus, and conveniences not to be easily had on board ship. It

may, however, be purified to a great extent, so as to become serviceable, by shaking it up in a good-sized bottle—a soda-water bottle, for instance—with a few spoonfuls of powdered white sugar. So far as the sugar is concerned, the action seems to be quite mechanical ; as its particles are shaken in amongst the mercury, they carry in with them quantities of air, the oxygen of which seizes on the tin or lead, forming an oxide round every particle of sugar, and leaving the mercury free. After shaking for a good time, the mercury may be poured off and strained ; but the process will have to be repeated two, three, or four times, with fresh sugar. Each time, after pouring off the mercury, a small quantity remains entangled, in exceedingly small globules, amongst the grey dirt that the sugar has changed into ; this may readily be washed out, falling to the bottom at once if water is shaken up with it, so that the dirt may be poured off ; it is as clean as the other, and may be mixed with it at once, so that all may go in together amongst the fresh sugar. In this way, as I have said, the mercury may be rendered serviceable ; but a good deal is lost, and the process takes some time and some trouble ; so that altogether it is much better to avoid the necessity for it, by taking care to keep the mercury clean.

The sextant should, of course, be a thoroughly good one ; and in choosing a sextant, great care and a due knowledge of the instrument are necessary.

As a general rule, it may be laid down that low-priced sextants are not worth much ; and, of the higher priced

ones, those that are built are better than those whose framework is in one piece ; the built ones do not perhaps look so nice, and some of them are rather heavy, but they are more likely to be true than the cast ones. A built sextant from a first-rate maker, whose name alone is a guarantee of the value of the instrument, cannot be had for a low price, and is worth a high one.

Amongst the particular points to examine into in choosing a sextant are—

1. The steadiness of the adjustments. The instrument ought to stand a fair amount of shaking—more even than in careful hands it is likely to be subjected to—without change.
2. The graduation marks on arc and vernier should be fine and clear, when examined through the microscope.
3. The vernier should lie, in all positions, with its edge perfectly close to the arc. Otherwise, if it is any little height above it, it is impossible to read correctly ; there is always an error of (what may be called) parallax, which is sometimes very considerable. This is a most important point to notice, and, in respect to it, low-priced sextants almost always fail.
4. The accuracy of the graduation. This may be tested by moving the vernier along the arc, making its zero mark successively coincide with divisions on the arc, and observing each time if the last mark on the vernier also coincides with a division on the arc. If it does not do so in every case, the graduation is incurably bad. Of course this can be repeated for every division along the whole length of the arc, but a few readings in

different positions will probably be sufficient, unless, for some other reason, the instrument be considered suspicious.

5. The telescope-ring, and the shades of the fixed reflector, are sometimes fitted so as to stop the moveable radius too soon, and thus curtail the readings at each end of the arc ; a sextant ought to read from 5° off to about 140° on, and though large angles are perhaps not quite trustworthy, it is as well to be able to take them when necessary.

6. The glass of the reflectors should be pure, free from air bubbles, streaks, or any such imperfections, and the two faces of each glass should be perfectly parallel to each other. Looking obliquely into each reflector at the image of some distant object will test this ; the image, as seen under every possible angle of reflection, should be clear and distinct, with sharp, well-defined edges.

7. The coloured shades should also have their respective faces parallel to each other ; very frequently they have not, even in otherwise good instruments. They may be tested by making an accurate contact of the sun's limbs, on or off the arc, as in taking an index error, using, in the first place, a coloured eye-piece ; and afterwards, in succession, the different shades in every possible combination or grouping. The readings off should, of course, be the same throughout, and any variation is due to the inaccuracy of the shades.

When this source of error does exist in a sextant, the index correction is different for every different pair of

shades, so that in observing it is necessary to note what shades are used.

The bright coloured shades, so generally fitted, are exceedingly trying, and indeed hurtful to the eye. Neutral tints are much to be preferred.

8. Lastly, the larger the sextant the better the graduation. The largest sextants made for ordinary sale are of 8 inches radius, but sextants of 10 inches radius have, I believe, been made to order. Held in the hand, the 8-inch sextant is rather heavy for long-continued observations ; a 10-inch sextant would be quite too heavy ; but for shore observations, where a stand can be used, decidedly—other things being equal—the larger the better.

Every one, whether navigator or surveyor, who has to work much with a sextant and to depend on it when quite out of reach of skilled workmen, ought to be thoroughly acquainted with its construction, so that he may be prepared, as far as possible, to devise a remedy should the instrument sustain any accidental injury. The best and perhaps only way of getting this knowledge is to practise on an instrument of no value, pulling it entirely to pieces, and putting it together again in proper adjustment. If in no other way, a sextant quite good enough for the purpose can frequently be picked up for a few shillings at a marine store ; and, in fact, such an instrument may be cleaned up, and tinkered a bit at the armourer's bench, or perhaps straightened out by a friendly engineer, the glasses resilvered, and the whole put into adjustment, so as to be capable of doing ordi-

nary work very satisfactorily. But it is as an instrument to practise on that I speak of it here ; so, after taking a few days' sights with it, for encouragement, the best thing to do with it is to take out the glasses and let it fall off the shelf on to the deck, or in some way or other subject it to very rough usage, and begin again to put it to rights. It is an amusement through weary hours at sea when there is nothing to do, and is very practically useful, into the bargain.

The most likely accident to happen to a sextant is the ruin of the silvering of the glasses by damp or wet. Sir Edward Belcher gives the following detailed directions for resilvering them, and as they have stood the test of many years' experience of many travellers, I prefer giving them in his own words.

'The requisites are clean tinfoil and mercury (a hare's foot is handy) ; lay the tinfoil, which should exceed the surface of the glass by a quarter of an inch on each side, on a smooth surface (the back of a book), rub it out smooth with the finger, add a bubble of mercury about the size of a small shot, which rub gently over the tinfoil until it spreads itself and shows a silvered surface ; gently add sufficient mercury to cover the leaf, so that its surface is fluid. Prepare a slip of clean paper the size of the tinfoil. Take the glass in the left hand, previously well cleaned, and the paper in the right. Brush the surface of the mercury gently to free it from dross. Lay the paper on the mercury, and the glass on it. Pressing gently on the glass, withdraw the paper. Turn

the glass on its face, and leave it on an inclined plane to allow the mercury to flow off, which is accelerated by laying a strip of tinfoil as a *conductor* to its lower edge. The edges may, after twelve hours' rest, be removed. In twenty-four hours give it a coat of varnish, made from spirits of wine and red sealing-wax.'

I have found it well to have the paper longer than is here said, so that it may be better taken hold of to draw out ; but it is no difficult matter, after a little practice, to do without it. When the glasses are set to drain, they should be placed with the edge which comes next the sextant downwards, for that edge will be downwards generally, that is to say, whenever the sextant is not in use ; looking-glass makers are particular on this point, and tell me that reversing a mirror is sure to cloud the silvering. The mercury which drains off is full of tin amalgam, and had better be thrown away ; it is useless for any other purpose, and will completely spoil the artificial horizon, if it should carelessly be poured in amongst the clean mercury.

It is a familiar fact which must be attended to, that a man will, as a rule, observe better with his own sextant, or one he is accustomed to, than with a strange one. For any work requiring great accuracy, such as that we are now speaking of, it should be mounted on a stand ; and on every occasion, before commencing the observation, the adjustments should be seen to. It is easier to adjust a sextant with a bright star than with the sun, and is not so trying to the eye ; but whether of sun or star, no observations should be taken without *first* seeing that

the reflected image will pass exactly over the direct one, and finding the index error.

The time should be taken by an assistant. For purposes of navigation, it is often convenient for the observer to take his own time, and this, for such ordinary work, is sufficiently accurate, but for rigorous observations on shore it is not so.

The only trustworthy observations for latitude are those of some heavenly body on or near the meridian; for these a star is preferable to the sun or moon; partly because its declination can be taken more exactly from the 'Nautical Almanack'; partly because refraction is less subject to troublesome vagaries by night than by day; but principally because under the same or nearly the same atmospheric conditions, two stars may generally be observed, one to the north of the zenith, the other to the south. By observing, to the north and south, stars of nearly the same altitude, at nearly the same time, and taking the mean of the two results, the personal error of the observer, and any error in the correction applied for refraction, are eliminated. This is, of course, not practicable when the sun is the observed body; and the single observations of him made, whether to the north or south of the zenith, do not give results so exact as the mean of two stars, taken as just described; neither can the result of the observation of the sun be meaned in this way with that of a star taken on the opposite side of the zenith, for the refraction at noon is certainly different from that during the night, and the personal error is probably very

different in the full blaze of sunlight and in the glimmering light of the stars.

When, however, from shortness of time, cloudy weather, or any other cause, it is necessary to trust to observations of the sun, the altitudes, with the time, should be observed continuously, from about a quarter before to a quarter after apparent noon ; the upper and lower limbs being observed alternately, so as to do away with the personal error, and with any error of semi-diameter arising from what has been termed *Irradiation*, or an increase of the apparent diameter by the intensity of the glow.

A star, in the same way, should be observed from about fifteen minutes before to fifteen minutes after its transit ; and another star of nearly the same brightness, of nearly the same altitude, but on the opposite side of the zenith, should be observed in exactly the same manner, as soon after as possible ; in each case observing carefully the barometer and thermometer, for the correction for refraction.

Of the various methods which have been proposed for reducing such observed altitudes to the meridian, none is so simple and so satisfactory as that given by Raper,¹ whose tables, indeed, render the approximate determination of latitude by these observations of the sun almost

¹ Here and elsewhere I have spoken of this as Raper's method, merely because it is by means of his *Navigation and Tables* that it is best known to naval officers. It was, however, I believe, first suggested by Delambre, and was used by him for the determination of positions in the great French triangulation towards the close of the last century.

as easy as that of actual meridian altitudes. The rigorous accuracy of this method cannot, of course, be used at sea, where the estimated latitude, the hour angle, and the observed altitude, are all necessarily more or less incorrect; but on shore, when the determination of the latitude to the nearest second is sought for, the exactness of Raper's method, as well as the simplicity of working it, comes prominently forward; and as the investigation of his rule is not contained in any of the usual text books of navigation, it seems not out of place to introduce it here.

Let l be the estimated latitude of the place of observation.

δ the declination of the heavenly body observed, considered negative if of the opposite name to the latitude.

z the observed zenith distance.

h the hour angle.

m the meridian zenith distance, that is, $l - \delta$; and less than the observed zenith distance by a small quantity known as the reduction to the meridian.

Let y denote this reduction to the meridian.

Then, applying the well-known cosine formula to the astronomical triangle,¹ we have

$$\cos z = \sin l \cdot \sin \delta + \cos l \cdot \cos \delta \cdot \cos h ;$$

or

$$\cos (m + y) = \sin l \cdot \sin \delta + \cos l \cdot \cos \delta \cdot \cos h.$$

¹ By the 'astronomical triangle' is understood the spherical triangle whose angular points are the pole, the zenith, and the observed heavenly body.

∴ expanding the first side of this equation,

$$\cos m. \cos y - \sin m. \sin y = \sin l. \sin \delta \\ + \cos l. \cos \delta. \cos h.$$

Substituting for $\cos y$ and $\cos h$ their values in terms of $\sin \frac{y}{2}$ and $\sin \frac{h}{2}$, this becomes

$$\cos m \left(1 - 2 \sin^2 \frac{y}{2} \right) - \sin m. \sin y = \sin l. \sin \delta \\ + \cos l. \cos \delta \left(1 - 2 \sin^2 \frac{h}{2} \right) \\ = \cos l. \cos \delta + \sin l. \sin \delta - 2 \cos l. \cos \delta. \sin^2 \frac{h}{2} \\ = \cos (l - \delta) - 2 \cos l. \cos \delta. \sin^2 \frac{h}{2}$$

$$\therefore \cos m - 2 \cos m. \sin^2 \frac{y}{2} - \sin m. \sin y = \cos m \\ - 2 \cos l. \cos \delta. \sin^2 \frac{h}{2}$$

$$\therefore 2 \cos m. \sin^2 \frac{y}{2} + \sin m. \sin y = 2 \cos l. \cos \delta. \sin^2 \frac{h}{2}$$

∴ dividing down by $\sin m$. and noting that

$$\sin y = 2 \sin \frac{y}{2} \cos \frac{y}{2}, \text{ we have,}$$

$$\sin y \left(1 + \cot m. \tan \frac{y}{2} \right) = \frac{2 \cos l. \cos \delta. \sin^2 \frac{h}{2}}{\sin m},$$

and as y is very small, the circular measure may be substituted for $\sin y$ and $\tan \frac{y}{2}$, without appreciable error :

$$\therefore y \left(1 + \cot m \cdot \frac{y}{2} \right) = \frac{2 \cos l \cdot \cos \delta \cdot \sin^2 \frac{h}{2}}{\sin m},$$

very nearly.

$$\text{Let } x \text{ denote } \frac{2 \cos l \cdot \cos \delta \cdot \sin^2 \frac{h}{2}}{\sin m},$$

$$\text{then } y \left(1 + \cot m \cdot \frac{y}{2} \right) = x,$$

$$\therefore y = \frac{x}{1 + \cot m \cdot \frac{y}{2}}$$

or, by actual division,

$$y = x \left\{ 1 - \frac{y}{2} \cot m + \&c. \right\}$$

$\therefore$ as a first approximation

$$y = x,$$

and as a second approximation

$$y = x \left(1 - \frac{x}{2} \cot m \right)$$

$$= x - \frac{x^2}{2} \cot m$$

$$= x - x'$$

if we denote $\frac{x^2}{2} \cot m$ by x' .

Further terms of the series, arising from the division, might be employed to give still closer approximations; but since they involve the cube and higher powers of x , which is a very small quantity, as depending on $\sin^2 \frac{h}{2}$

or $\left(\frac{h}{2}\right)^2$, and since $\cot m$ is never very great, that is to say, that a star near the zenith cannot be observed with the artificial horizon, these later terms are so small as to be of no consequence whatever; even x' , the correction for the second approximation, has scarcely an appreciable value unless h is greater than fifteen minutes, after which its value increases more rapidly.

We have, then, as a first approximation,

$$y = x;$$

or, putting the result into seconds of arc,

$$y \text{ (in seconds)} = \frac{x}{\sin 1''}$$

$$\begin{aligned} \text{and} \quad \log y &= \text{Log cosec } 1'' + \log x - 10 \\ &= \log 2 + \text{Log cosec } 1'' \\ &\quad + \text{Log cos } l + \text{Log cos } \delta \\ &\quad + \text{Log cosec } m + \text{Log hav } h - 50, \end{aligned}$$

and for the second approximation

$$\log x' = 2 \log x + \text{Log cot } m - \log 2 - 10,$$

from which x and x' being determined, as before,

$$y = x - x'.$$

For the first approximation, Raper tables,

$$\{\log 2 + \text{Log cosec } 1'' + \text{Log hav } h - 20\}$$

for successive values of h ; but this is useful principally for working out isolated examples. When, as in fixing the latitude of a station, a very great number of examples have to be worked out with the same latitude, and even with the same star, that is, with the same declination, and

same meridian altitude, the work can be much simplified by finding

$$\{\log z + \text{Log cosec } 1'' + \text{Log cos } l + \text{Log cos } \delta \\ + \text{Log cosec } m - 40\}$$

which is a constant log for all observations of that particular star at that place; and the reduction to the meridian in the first approximation is at once found by adding to this constant log, $\{\text{Log hav } h - 10\}$, which gives $\log x$.

This simplification is indeed necessary, or the work would be of most formidable length, as each single observation ought to be worked out separately, and the number of observations of the one star made in the half-hour, on one night, will probably be between 20 and 30. It is very usual to take these observations in sets of *three* or *five*, and to consider the arithmetic mean of a set as of greater value than any single observation. This idea is quite erroneous, for it is founded on the supposition that near the meridian the reduction varies as the hour angle, that is

$$y \propto h,$$

whereas we have seen that in the first approximation, which alone is of any consequence,

$$y = z \frac{\cos l \cos \delta}{\sin m} \sin^2 \frac{h}{2},$$

that is $y \propto \sin^2 \frac{h}{2};$

or, since $\frac{h}{2}$ is very small,

$$y \propto h^2 \text{ nearly.}$$

The compounded observation is thus not so correct as a single one, and though the error arising from meaning the observations in this way is not very great, it may amount to 5'' or more, and is especially objectionable, as it is a perfectly unnecessary error, wilfully introduced on a false principle.

Each single observation being therefore worked out separately, which, by the method just explained, is no very formidable piece of work, the arithmetic mean of the reduced zenith distances is taken, from which the latitude is found as by an ordinary meridian altitude. The latitude is again found, in precisely the same manner, from the arithmetic mean of the reduced zenith distances obtained from the second set of observations. But the mean of these two values of the latitude is not simply the arithmetic mean, unless the number of observations of each star is exactly the same, for the latitude derived from the greater number of observations is more to be depended on than the other. If, for instance, the arithmetic mean of m observations of a star north of the zenith gives a latitude l_1 , and the mean of n observations of another star south of the zenith gives a latitude l_2 , then if m is greater than n , l_1 is more to be depended on than l_2 , in the ratio of $m : n$; and the true mean is

$$\frac{m l_1 + n l_2}{m + n}.$$

As the difference between l_1 and l_2 can scarcely be more than a few seconds, the application of this formula is simple enough; for the seconds only, or perhaps only

the units of the seconds, require to be multiplied, added, and divided. Thus, suppose that

30 observations to the north give a latitude $32^{\circ} 27' 36''.5$
and

20 observations to the south give a latitude $32^{\circ} 27' 47''.3$

the mean of the seconds is $\frac{36.5 \times 30 + 47.3 \times 20}{50}$ or $40''.82$,

and the mean latitude is $32^{\circ} 27' 40''.82$.

Unless the difference between the two latitudes is large, and also the difference between the number of sights in each set, the error arising from using the arithmetic mean of the two is small ; but, if there is any considerable difference and inequality, it may be very noticeable ; and, in any case, we have no right to introduce avoidable errors in order to escape a little multiplication and division.

If two pairs of sets are taken on the same night, they may all be meaned at once in the same way ; that is,

the mean of the four sets is $\frac{m l_1 + n l_2 + p l_3 + q l_4}{m + n + p + q}$,

m , n , p , and q being the number of sights in each set of observations. For the final result, the result of the several nights' work should be meaned in exactly the same way ; and if the observations have been running on for six or eight weeks, in good weather, so that one or two pairs of 'sets' of 20 or 30 sights have been taken nearly every night, we may consider that the latitude of that particular spot has been determined with all the accuracy that the sextant admits of.

The various methods of finding the absolute longitude of any place, by direct reference to a prime meridian, theoretically beautiful as many of them are, give, in practice, results which are vague and frequently even wild in the extreme. Observations of eclipses of the sun or moon—a rare class of phenomena—of the eclipses of Jupiter's satellites, and of occultations of stars by the moon, cannot be made with any exactness without stationary telescopes of a very high power; and the observations of lunar distances with the sextant cannot, as is familiarly known, be depended on as giving results correct within *fifteen or twenty minutes* of arc. It is true that in the case of lunars this large error may be reduced by taking the mean of two observations, one of a star to the east, one of a star to the west of the moon, thus probably neutralizing the errors of observation; and by taking a very great number of observations, both east and west, continued through a period of two or three lunations, the longitude may be determined to perhaps the nearest minute of arc; but the result is not sufficiently satisfactory as at all to compensate for the enormous amount of labour spent not only in taking the observations but in computing them afterwards. In fact, although lunar observations, in pairs east and west, afford a most valuable check on the chronometers during a long voyage, and more especially during a first voyage, when the rates are perhaps not very exactly known, they cannot be considered of any use for purposes of surveying, or determining a position with rigid accuracy.

The only way, then, in which longitude can be satis-

factorily obtained is by chronometric observations ; but the longitude so found cannot properly be referred to the prime meridian, for its accuracy would then depend, in a very great measure, on the error of the chronometer on the time of the prime meridian, and this cannot possibly be determined with certainty. All longitudes of positions on shore are, therefore, in the first instance, referred to some neighbouring meridian, which, passing through some convenient, well-known, and frequented place, is used as a secondary meridian, and are referred afterwards to the prime meridian by applying the accredited longitude of the position of reference. The care of the surveyor thus extends only to determining his longitude east or west of whatever secondary meridian he may refer to ; and, so far as his more immediate work is concerned, leaves entirely on one side the consideration of any possible error in the longitude of the place of reference. Such an error in no way affects the relative longitudes which he seeks to establish ; and these being determined and registered, the absolute longitudes which are put on the charts, or in tables of position, can at any future time be reduced for a correction in the longitude of the station of the secondary meridian. It seems, indeed, probable that, with the present extension of the electric telegraph, the absolute longitude of many stations will, in the course of the next few years, be determined with an accuracy hitherto unattainable ; but even then, with a view to the possible discovery of entirely new methods, as yet undreamt of, and still more certain in their results, it will remain the duty of the surveyor to

work ostensibly, as well as in reality, from a secondary meridian.

So far, therefore, as relates to purposes of surveying, the longitude of a position is the distance, whether in time or arc, between its meridian and the meridian of reference ; and the immediate meridian of reference may be either the meridian of some well-known spot, which has been accepted as a secondary meridian, or it may be the meridian of any position already determined in the course of the survey ; or, indeed, it may be the meridian of some place not yet determined, but which may be determined hereafter. It is the difference of longitude, that is, the meridian distance between the two places, which is more especially sought for, and can be most accurately found ; and, on this account, it is more usual, in surveying, to speak of determining a ' Meridian Distance ' than of determining longitude.

The management of chronometers, and the method of comparing them, and of determining their errors and rates, belong so strictly to the ordinary duties of the navigator, that it would be out of place to enter on a full discussion of them here, although a thorough knowledge of them is in the highest degree necessary to the surveyor.

The best way of finding the error of a chronometer is by the observation of equal altitudes ; and though equal altitudes of the same star offer some advantages over those of the sun, as, for instance, the simpler nature and greater theoretical accuracy of the computation, owing to there being no change in declination, practically these



advantages are more than counterbalanced by the inconvenience of having to make night observations at two widely different times, and by the fact that it is frequently impossible to make the observations at all, in consequence of no suitable star being on or near the prime vertical twice during the same night ; but by care in observing the sun, noting the barometer and thermometer at the time and place of observation, the accuracy of the result is scarcely, if at all, inferior to what would be attained by the observation of a star under the most favourable circumstances.

Different observers recommend different ways of taking these observations ; that which, on the whole, seems preferable is to take them in *pairs*, in this manner—

Set the sextant to some convenient degree, a little greater than the altitude of the upper limb, in the forenoon, when the sun is rising. Observe carefully the time of contact of the upper limb ; and, without altering the set of the sextant, observe also the contact of the lower limb, a few, that is about three, minutes later.

The mean of this pair gives a result comparatively free from personal error, or from error in semidiameter, whether from irradiation, or other causes. In the afternoon, of course, the order is reversed ; the sextant is set to the same reading, and the times of contact observed, first of the lower limb, and, about three minutes later, of the upper limb.

It is not unusual to take the mean of several such pairs of observations, and to work with it as with a single observation. It is more strictly accurate to find the error of a

chronometer separately from the mean of each pair, and to take the mean of the results as the true error.

When the error has been found in this way on two days, with seven or eight days between, the difference between the two errors, divided by the number of days, gives the rate, on the supposition that the rate is uniform ; and unless there is time for further observations, we are practically dependent on this supposition, although we know that chronometers very rarely do keep an uniform rate, even when they are just out of the maker's hands. If time permits further observations, the successive determination of the rate from week to week will give a tolerably correct estimate of the change to which the rate is subject.

If the chronometers are good, and meet with no unfair treatment, changes of temperature are almost the only cause of irregularity in their rates, and these irregularities are often extremely difficult to guard against. The temperature at sea is generally so different from that in harbour, that the rates at sea do, as a rule, vary considerably from those determined in harbour ; and in making a passage, no opportunity of verifying them should be lost. Lunar observations afford a valuable check, and would detect any gross error ; carefully taken, in pairs east and west, they may be depended on as being within thirty seconds, and probably less, and the dependence will be much closer if the observations can be repeated several times, and carried on through two or three nights. If well-known land should be sighted at a convenient distance, an easier and more certain method



is to fix the position of the ship by cross-bearings and find the error of the chronometer on that position, by single altitude of the sun ; with proper care, and under favourable circumstances of weather, this may be depended on to within a few seconds, and will give a corroboration or correction of the rate that may to a certain extent be relied on ; but in estimating the value of this, the navigator must take into consideration the changes of climate which the ship has passed through, and the possibility of the difference between the newly determined and the computed error having arisen from a change of rate during a part of the time only.

To a surveyor, sighting unknown or imperfectly known land, no such opportunity is offered ; on the contrary, it is probably his duty to fix the position of such land as closely as time will allow ; if possible, by observations on shore ; if not, by observations at sea, from a spot fixed as before either by cross-bearings or by other methods which, as well as their reduction and exact application to the present problem, will be discussed on a later page. If the observations are made on shore, so that the error of chronometer on the local time may be determined with reasonable accuracy, then the meridian distance, that is, the difference of longitude from the port quitted, is at once found approximately by applying the error accumulated from the rate determined before sailing, and is afterwards found more correctly by a comparison of rates subsequently determined. But if the ship can stay long enough to determine the rate of the chronometer there, then the rate during the interval

between the two determinations may be estimated as the mean of the two; the result being more or less trustworthy according to the goodness and number of the chronometers.

Rules of this nature are often more clear if put into an algebraic form; and as some of the more exact reductions—to the explanation of which I am now proceeding—require an algebraic notation, I may as well introduce it at once.

Let a denote the daily rate
 α denote the whole error } of the chronometer, as
 determined on quitting port A .

b the daily rate
 β the whole error } on arriving at port B .

M the meridian distance between A and B .

t the number of days on the passage.

Then, in the first case just mentioned, when the error only, not the rate, has been determined at B , we have approximately,

$$M = \alpha + t\alpha - \beta,$$

B being to the east of A if this expression is positive, and west of A if it is negative.

But if, as in the second case, the rate also at B has been determined, then

$$M = \alpha + t \frac{\alpha + b}{2} - \beta,$$

a result on which greater reliance can be placed.

If at intervals $t_1, t_2, t_3 \dots$ days from leaving A , the ship touches at places $B_1, B_2, B_3 \dots$ and determines

only $\beta_1, \beta_2, \beta_3 \dots$ the errors on local time, and, as before, on arriving at B determines the rate b , the rates at the end of $t_1, t_2, t_3 \dots$ days not being observed, have to be calculated, which may be done by ordinary proportion. Let $b_1, b_2, b_3 \dots$ denote these rates, and the question is,

If in t days the rate change from a to b , what would it change to in t_1 or in t_2 or in t_3 days?

In t days the change is $b-a$, being positive if the rate is increasing and negative if decreasing.

In t_1 days the change is b_1-a , and we have the proportion,

$$t : t_1 :: b-a : b_1-a,$$

whence
$$b_1-a = \frac{t_1}{t}(b-a)$$

and
$$b_1 = a + \frac{t_1}{t}(b-a).$$

Hence we have the mean rate during the interval t_1 ,

$$\frac{a+b}{2}, \text{ or } \frac{1}{2} \left\{ 2a + \frac{t_1}{t}(b-a) \right\}$$

and the accumulated error for the same interval is

$$t_1 a + \frac{t_1^2}{2t}(b-a);$$

this gives then the meridian distance of B_1 from A ,

$$M_1 = a + t_1 a + \frac{t_1^2}{2t}(b-a) - \beta_1$$

and similarly for the other places $B_2, B_3 \dots$

Attempts have been made to improve these formulæ by the introduction of a correction for temperature. Such a correction would be a very decided step in advance, but as yet nothing sufficiently definite has been proposed, or has stood the test of actual trial ; and, in fact, in the absence of satisfactory experience, the methods which have been proposed differ so much amongst themselves that the practical mind recoils from the idea of putting confidence in any of them. Although, therefore, the study of them is interesting, and may lead to profit at some future time, at present it is a matter which can be considered only theoretical, the discussion of which would lead us far beyond the scope of the present work. All officers who have the charge of chronometers establish certain empirical rules of their own, as to the effects of changes of temperature, in the formation and application of which they are guided by their experience and their intimate knowledge of the peculiarities or vagaries of the several chronometers under their charge. In this, an experienced navigator will find little difficulty ; and an inexperienced one is scarcely likely to be called on, except as a mere exercise, to determine a Meridian Distance in which any such correction is required.

Nevertheless, the determination of Meridian Distances cannot be considered *entirely* satisfactory so long as the rate and the change of rate enter into the formula. In practice they can be avoided whenever the meridian distances are not too great, and can be measured both ways. When this can be done, and the opposite runs are made within a short time of each other, so that the

change of rate is inappreciable, and the sea rates, during passages performed under similar conditions, may be properly assumed to be equal, we obtain a very simple expression for the Meridian Distance, free from all error arising from error in rate.

Let the run from *A* to *B* occupy *t* days, and the return passage from *B* to *A* occupy *t'* days. Let also the error of chronometer on local time

on leaving *A* be α ;

on arriving at *B* be β ;

on leaving *B* be β' ;

on arriving at *A* be α' ;

and let *x* be the sea or passage rate.

Then from the first run, we have

$$M = \alpha + tx - \beta$$

and from the second run,

$$M = \alpha' - t'x - \beta' ;$$

from which two equations we find

$$x = \frac{M - \alpha + \beta}{t},$$

and
$$x = \frac{\alpha' - \beta' - M}{t'} ;$$

∴ equating these two values of *x*,

$$\frac{M - \alpha + \beta}{t} = \frac{\alpha' - \beta' - M}{t'} ;$$

$$\therefore M(t + t') = t'(\alpha - \beta) + t(\alpha' - \beta'),$$

and

$$M = \frac{t'(\alpha - \beta) + t(\alpha' - \beta')}{t + t'},$$

an expression in which the rate, either in harbour or at sea, does not appear.

The errors being determined by equal altitudes, the *times* necessarily are counted from noon before each departure to noon after arriving. If the distances are short, there is thus no difficulty in arranging so that the times of the two passages may be the same ; in which case $t' = t$, and the formula reduces to

$$M = \frac{1}{2} \{ (\alpha - \beta) + (\alpha' - \beta') \}$$

and in this form it is of great practical use.

If the ship, shortened in and with steam up, is ready to weigh at once, as soon as the officer comes on board from taking the afternoon sights, she leaves her anchorage by 5 P.M. at the latest, and by 7 or 8 A.M. the following morning may easily have run a distance of 100 or 120 miles ; so that at any place within this distance, the officer in charge of the observations arrives in time to take the forenoon sights ; and the errors α and β are determined on consecutive days, thus affording the shortest time for any improbable but still possible irregularity. The return passage can be made in the same manner ; and a meridian distance, determined by observations so conducted, and by the formula just given, being free from errors of rate, known or unknown, and from chances of accidental jumps or irregularities, is sufficiently satisfactory. As often as practicable during the course of the survey, the double passage and sets of observations should be repeated ; and the mean of several

such determinations will give a result as accurate as can be obtained with the instruments used.

In dwelling thus on the necessity of long-continued observations to determine the latitude and longitude with the greatest possible correctness, it must still be remembered that a highly respectable degree of precision can be attained in a short time, when necessary. If only one day can be spared for the observations, equal altitudes of the sun taken in pairs, in the manner already described, will satisfactorily determine the error of the chronometer on local time ; but if the ship cannot stay for the afternoon sights, or arrives too late for the forenoon, the error of the chronometer must be determined by single altitudes, which, as all altitudes of the sun, should be taken in pairs, upper and lower limb. Observations of the sun near the meridian, taken as described, upper and lower limb alternately, will give the latitude, to a very close approximation. If the sun's altitude near the meridian be so great that it cannot be taken in the artificial horizon, and the ship cannot remain till night, all that can be done is to take some sets of double altitudes, by which the latitude may be found to perhaps the nearest minute ; but this method is rough and unsatisfactory, and should never be employed except in cases of absolute necessity. If the ship arrive in the evening and stay till morning, a busy night may determine both latitude and longitude with very great accuracy ; the observations for latitude being conducted in exactly the manner already dwelt on, sets being taken of every suitable star that passes the

meridian ; and though equal altitudes of stars are not always possible, the results of independent observations of different stars, at nearly the same altitude, and near the prime vertical, east and west of the meridian, will give a mean almost as trustworthy. But it must be remembered that the results of these hurried and abridged observations are merely approximations more or less rough ; we cannot expect anything like an exact determination ; all that can be aimed at is to do the best that is possible under the circumstances.

I have here spoken especially of one of the extremities of the base line as the point where latitude and longitude is to be astronomically determined. It happens, however, sometimes, and more commonly in an extended survey, that an extremity of the base line is a very inconvenient place at which to make these long-continued observations ; that there is no sufficient length of level ground, to admit of measuring a base in the immediate neighbourhood of the observing station ; and that it is considered essential to determine astronomically either the extreme points embraced by the survey or some other point of evident geographical importance.

When this is the case, the latitude and longitude of either end of the base line are deduced from those of the point so determined by a method somewhat analogous to dead reckoning, trigonometrically conducted on shore, and known as the triangulation ; the details connected with which I shall presently have to discuss at some length.

The long continuance of the observations for latitude and longitude need not in any way delay the other work of the survey. As soon as the stations have been marked, the base line should be measured. This measurement is actually made, either with a chain, a cord, or a rod of known length ; but whatever is used, very great care is necessary.

In the more exact work of the principal land surveys, such as the ordnance survey of England or Ireland, or the great trigonometrical survey of India, these measurements are made with compound bars, having an ingenious self-acting compensation for temperature ; these bars rest on carefully levelled supports, and overlap each other at the ends, where they are finely graduated, and are fitted with sliding verniers which, read by microscopes, and moved by tangent screws, admit of the most perfect accuracy in bringing them together. A measurement conducted in this way is a work of very great trouble and expense, and cannot possibly be repeated at frequent intervals in different places. For the purposes of nautical surveying, ruder measurements have to suffice, and anything like perfection is not to be hoped for. But in this and every other point connected with nautical surveying, it will comfort the surveyor to remember that his work when completed is for the guidance of those who, in following it, will use instruments certainly no better than those which he himself has used, and without his chances of being exact. The surveyor may determine his latitude and longitude till it is a question of seconds or tenths of seconds ; or his linear distances, till a few feet

seem of importance ; to the seaman using the chart, such degrees of accuracy are not possible, nor are they cared for.

The chains which are now supplied to surveying ships for these measurements are of steel ; they are 100 feet in length, each of the long links with the intermediate short ones being one foot ; they are carefully made for the purpose, and are as exact as such things can be. Similar chains used in the land surveys, before the invention of the compensating bars, had links two and a half feet long ; but links of such a length, though offering less chance of error from stretching or overriding, are inconvenient on board ship, or in boats, and would run great risk of being bent or broken.

The so-called Surveyor's or Gunter's chain of sixty-six feet is but little used, except for field work, and for that, the readiness with which its measurements can be reduced to acres and roods renders it peculiarly advantageous.

In order, whilst measuring, to keep straight between the two stations, numerous smaller poles or staffs bearing little flags must be set up along the line ; the pole at the first station is removed, and the theodolite set up in its place, care being taken to bring its centre exactly over the centre of the hole, by means of the attached plumb-line ; through the telescope it is at once seen if the minor flag-staffs are in true line ; if not, they can be moved, right or left, according to the observer's signals. Some authorities recommend that in addition a guiding cord should be stretched along from end to end, bearing

against each of these flag-staffs, and always on the same side of them ; but if the measurement is being made with chain or cord, it is scarcely necessary, as the chain-bearers can easily keep themselves pretty straight by the flag-staffs, and can be checked by the observer at the theodolite. With the rod, such a cord ought to be used, as it can never cover the distance from one flag-staff to another.

In measuring with a chain, the one end is placed exactly under the plumb line of the first station, and the other end carried forward ; when the leader has reached the full stretch of the chain, in the proper line, he drives in a small iron picket close up against the foremost end, and then moves on ; as the follower arrives at this picket, he holds the hinder end of the chain against it, whilst the leader at the other end drives in a second picket ; the follower then pulls out the first picket, which he carries with him, and proceeds, as before, to the second picket. In this way they go on till the second station is arrived at. The length of chain which reaches from the last picket to the station is known by counting the long links, one foot for each, and measuring the odd inches, if any. It is usual to have only ten pickets ; and when these have been all driven in, the follower, who has collected them, returns them to the leader, making a note of them in the sight-book ; each entry thus stands for 1,000 feet ; the pickets which he has in hand at the end for 100's ; and the odd links and fraction of a link for feet and inches.

The measurement so taken should be repeated at least

two or three times—oftener if practicable ; the results will probably, indeed almost certainly, differ much more than one would at first expect. With every possible care in conducting the operation, the trifling inequalities of the ground must cause some error ; as this error tends to make the measurement too great, on the other hand, the thickness of the pickets, and any slight stretching of the chain, tend to make it too little ; any overriding of the links—which can only occur through carelessness on the part of the chain-bearers, and especially of the leader—would, again, make the measurement too great ; but as these sources of error, to some extent, counterbalance each other, the resulting error is probably not very serious. That arising from the inequalities of the ground is the most important ; and as this *always* tends to make the measurement too great, it is perhaps a question whether the *least* measurement made is not more nearly accurate than the mean of all. It is, however, a dangerous principle to admit that any one observation is better than another made with equal care ; and in this case the mean of as many measurements as can be made has probably no very great error. So much depends on the smoothness of the ground that it is impossible to lay down anything like a limit to the difference which may exist between the observations ; one foot in 500 is perhaps as little as can be hoped for, under the most favourable circumstances ; but even if it should be a great deal more than that, it is not of much consequence, and will, to a great extent, disappear in the mean.

A chain, such as I have spoken of, is not commonly

supplied except to surveying ships ; when it is necessary to undertake a small piece of surveying work without one, a cord must be used ; and nothing is better suited for the purpose than a piece of old well-stretched deep-sea line. When it is likely to be often wanted, it would be well to have it cut to the length of 100 feet, and kept specially for the purpose ; it should, nevertheless, be carefully measured (as a log-line is measured) by marks on the deck, whenever it is to be taken on shore for use. If each end of the line is turned round a thimble, a pair of handles (which can easily be made by the blacksmith) may be fastened to these thimbles by independent lacings, which admit of being readily shortened in, or lengthened out, so as to adjust the whole length to exactly 100 feet whenever it is compared. It is then used exactly as the chain, the only difference being that the odd feet from the last picket to the second station have to be measured separately, there being no links to count ; but pieces of coloured wool may be drawn through at every five feet, which, though changing slightly with the stretching or shrinking of the line, will remain pretty accurate, and not introduce any error worth noticing.

A rod of some twenty or thirty feet long seems to have been preferred by the older surveyors ; but the short length makes it very troublesome, and though apparently more correct than the cord, it is difficult to manage over ground at all rough, and is exceedingly likely to cause mistakes. When a chain is not to be had, the cord is both safer and easier to use than the rod.

In selecting the stations for the extremities of the base line, it may often be convenient, and indeed necessary, to choose positions between which actual measurement cannot be carried out, as for instance when small outlying rocks or islands form the only available sites for one or both of the stations. This may especially be the case when the coast-line to be surveyed is nearly straight, with difficult or thickly wooded country behind, and generally when the tract of low land near the coast is of very limited extent, or when the coast is altogether rugged and rockbound. When such stations have been fixed upon, the distance between them can only be measured by *sound*, that is, by noting the time that sound takes to travel the distance, and comparing it with its known velocity. Such a method is not so exact as actual measurement; nor indeed can it be, for since the velocity of sound is more than 1,000 feet a second, a mistake in noting the time of even $\frac{1}{10}$ th of a second would correspond to an error in distance of more than 100 feet; besides this, the velocity of sound, in all conditions of the air, is not known with perfect exactness; it varies a good deal both with temperature and humidity, and the changes dependent on humidity have never been satisfactorily ascertained. It is also seriously affected by wind, and as wind is at all times very irregular in its motion, even the mean of a number of observations can seldom be quite correct. It is, however, the only way of measuring the line, when it is necessary to choose such stations, and by care and exactness in the observation, the length can be

determined with a much nearer approach to accuracy than would at first sight seem possible.

It is evident that the relative value of any error arising from an error in the observed interval will be less, according as the interval itself is greater ; that is, according as the line is greater. The stations should therefore be selected as far apart as practicable, and unless absolutely necessary the line should not be less than a mile and a half ; from two to three miles is frequently a convenient length. The observation consists in noting at one station the time which elapses between seeing the flash and hearing the report of a gun fired at the other station. A gun of suitable size is taken to one station, and a signal made, as by hoisting a flag, when all is ready ; this is answered from the other station, and the gun is fired a couple of seconds after the flag is hauled down, or otherwise, as may be agreed on ; the observer, with a watch, at the other station, as he sees the flash, counts the seconds and the fraction of a second till he hears the sound ; at the same time, the temperature is noted. This should be repeated a great number of times.

Such an observation should not be attempted on a windy day ; but even a light air may produce a considerable effect, and occasional puffs during the progress of the observation may cause some irregularity ; if therefore any of the single observations is markedly different from the others, and the difference can be attributed to a freshening of the breeze for the moment, such a one might properly be rejected from the mean ; but to avoid the possibility of a similar error affecting the whole of

them, and thus the mean itself, the observation should be reversed; that is, the gun and the watch should change stations, and the observation be repeated as before.

The mean of each set of observations is then found in the usual simple manner, adding them all together and dividing by the number of observations; but the final mean, or the mean of these two means, cannot be found in the same way, and for this reason, that if there is any difference between the two, it is due to the wind acting along the line, and the sound has been less affected by it during the shorter time than during the longer; it has therefore been retarded when going against the wind more than it has been quickened when going with it. The simple arithmetic mean of the two, founded on the supposition that it is retarded by the one as much as it is forwarded by the other, will thus clearly give the interval too large.

The mean required has to be found in this way :

Let d denote the distance in feet, between the stations,
 v the velocity of sound, } both in feet per second,
 x the velocity of the wind, }
 t_1 the observed interval, in seconds, *with* the wind,
 t_2 the observed interval, *against* the wind,
 t the true interval required.

Then $d = vt,$

but by observation $d = vt_1 + xt_1,$

and $d = vt_2 - xt_2 ;$

hence we have

$$\frac{d}{t_1} = v + x$$

$$\frac{d}{t_2} = v - x$$

∴ adding, we get

$$d\left(\frac{1}{t_1} + \frac{1}{t_2}\right) = 2v$$

or

$$d = v \frac{2t_1 t_2}{t_1 + t_2}$$

but we have also

$$d = vt$$

∴ equating these two values of d we get

$$t = \frac{2t_1 t_2}{t_1 + t_2},$$

that is, in words, the mean interval required is obtained by dividing twice the product of the first mean times by their sum.

The mean interval multiplied by the velocity of sound gives the distance.

The result of many careful experiments shows that the velocity of sound is 1,090 feet per second, when the thermometer is at 32° Fahr., and that it increases as the temperature increases, at the rate of $\frac{1}{8}$ ths of a foot for each degree. We may thus table :

Thermometer Shows	Velocity
32	1,090
41	1,100
50	1,110
59	1,120
68	1,130
77	1,140

and the velocity for any particular temperature may be taken out by inspection.

An example of this method of finding the length of a line may perhaps render it more clear. Suppose that the mean intervals, as given by each set of observations, are 11.5 and 12.3 seconds; and the mean reading of the thermometer is 63° Fahr.

The nearest degree in the table to 63° is 59°, and the correction for 4° is $4\frac{4}{5}$ of a foot, or 4.5 very nearly. This gives the velocity of sound at the observed temperature 1124.5.

The time interval, according to the rule already laid down, is $\frac{2 \times 11.5 \times 12.3}{11.5 + 12.3}$, which reduces at once to

$$\frac{11.5 \times 12.3}{11.9}.$$

Hence we have the distance $\frac{11.5 \times 12.3 \times 1124.5}{11.9}$ which can be reduced easily enough by means of logarithms, and gives the required length of the line 13,366 feet.

The direction of the base line is determined by determining the bearing of the second station from the first. This must not be done by the compass. As a general rule, the compass is never used in any surveying work where accuracy is required, as its deviation cannot be depended on. There is always a chance that the ground on which it happens to be placed may contain iron in some form or other; and the local deviation of a compass on shore is excessively capricious, varying through as

much as a quarter of a point each way, and at times a very great deal more, without any apparent reason. On this account, all bearings are calculated from angles taken with the theodolite or sextant. With the theodolite this is extremely simple; take the horizontal angle between the other station and the sun, and this, referred to the true bearing of the sun, at the same instant, which is found in the usual way either by observing the time, the altitude, or both, according to the choice of the observer, gives at once the true bearing of the station required. But when no theodolite is available, the angular distance between the sun and the station is observed with a sextant; this is obviously not so convenient, as the difference of bearing has to be calculated from the quadrantal triangle, whose angular points are the station, the zenith, and the sun.

The bearing of any other point, from the initial station, is at once known by referring it to the bearing of this second station, just found, adding or subtracting, as the case may be, the horizontal angle between the second station and the point whose bearing is required. This horizontal angle is taken immediately with the theodolite; but when the sextant is used, and the point observed has any altitude, this also must be taken, and the oblique angle between the two points has to be reduced by the solution of the triangles whose angular points are the two stations observed, and the zenith; the angle at the zenith is the horizontal angular distance required. In this particular case, the triangle is quadrantal, as the second station has been supposed to be on the same

level as the first ; but when bearings are being taken in a similar manner from other stations, and both the observed stations have altitudes, the triangle is no longer quadrantal, and the reduction is consequently slightly longer.

There may be some difficulty about taking the altitude of a distant point, such as the summit of a hill, with the sextant. A little practice will teach the observer to hold the instrument with the line of sight approximately horizontal ; and if greater accuracy is required, some distant spot, a stone or a bush, on the horizontal line, and directly underneath the summit, may be picked out by means of the water, or, as it is sometimes called, the French level. To this spot the peak may be brought down without any further difficulty ; or if by chance no such spot should be found, the peak can easily enough be brought down to the water line of the level itself.

Similarly, when very great accuracy is not required, the horizontal angle, or difference of bearing between two points of different altitude, may be taken at once with the sextant, by setting up a plumb-line 'on' with the left of the two objects, at some little distance from the eye, holding the sextant as nearly horizontal as can be judged ; the reflected image of the right hand object may then be brought to the plumb-line, seen direct, and swept along it up and down, in exactly the same manner that the image of the sun is swept, right and left, along the line of the sea horizon, till a correct contact is made. This, of course, saves a good deal of trouble in the reduction, and is sufficiently accurate for most purposes.

The true bearing of the second station having been determined, as well as the length of the base line, and the latitude and longitude of the initial position, the line can be laid down on the chart, according to its scale, and the determination of other stations can be proceeded with.

The primary stations must be marked with poles and flags, in the same manner as those of the base line ; these stations are chosen, partly as important points in the survey, which it is desirable to fix accurately, but principally by their relative position with respect to the ends of the base line, or to each other. The first use of the stations is to serve as angular points of triangles ; and the particular triangle of which any one has to form an angular point must be the first consideration in its selection. It must therefore be borne in mind that no sufficient dependance can be placed on any triangle which is widely different from equilateral ; in protracting two lines, it will at once be seen that if they cut each other at a small angle, the point of their intersection is not at all well defined, so that, even on a large scale, a very serious error might easily be made in the position. The clearest cut is evidently when the two lines are at right angles to each other ; the best cut available for the sides of a triangle is at 60° , and no triangle should be admitted into any system of triangulation which has an angle less than 30° . Triangles which deviate so much from the equilateral form as to be inadmissible, are usually denominated *ill-conditioned triangles*.

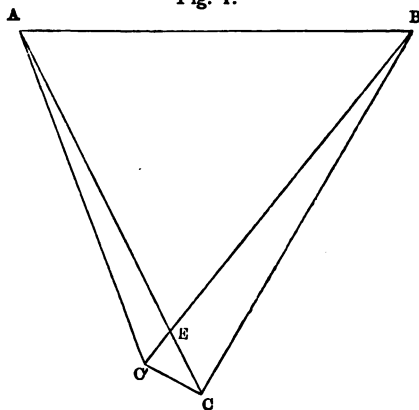
As soon as the poles have been set up at the several

stations, the observer should begin taking the angles between them. With the theodolite, if he has one, or, if not, with the sextant, he should, at the first station, take the angles between the second station and *every* station, or, indeed, every prominent object that is visible. Moving to the second station, he should then take the angles from the first station to every other, and proceeding to the next station, should in the same manner take a complete round of angles, so as to measure the angle subtended by every pair of visible stations. In this way, all the stations should be gone through ; a complete round of angles being taken at each, so that every station may be determined by the agreement of as many independent observations as possible. At each station the pole should be temporarily removed, in order that the theodolite may be set up in its exact position ; if this is not done, the observations ought to be reduced ; and, on this account, the plan of marking a station with a dash of whitewash on a rock, a boulder, or on a tree, is inaccurate, or troublesome, although in a hurried survey it may be convenient to adopt it. But it is sometimes necessary to use such objects ; or, after the observations have been taken at a station, it may have been thought advisable to mark it permanently with a large cairn built up round the pole ; if further observations have to be taken at that station, the theodolite must evidently be set up in the neighbourhood, and the observations ought to be reduced in this way.

Let C be the true station, and C' the station where it has been necessary to set up the theodolite. To find the *angle at C between two stations A and B* , we observe

at C' the angle $AC'B$; also the angles $AC'C$ and $BC'C$; and measure the distance CC' .

Fig. 1.



AC and BC are known approximately from other parts of the triangulation, or can be found with sufficient accuracy by finding AC' , BC' and allowing the difference by estimation. The side CC' , is so small in comparison to these other sides, that any possible error in this estimation does not appreciably affect the result.

Now the angle AEB is the exterior angle of both the triangles BEC and AEC .

We have therefore,

$$\begin{aligned}\angle E &= \angle C + \angle B \\ &= \angle C' + \angle A\end{aligned}$$

$$\therefore C + B = C' + A$$

and

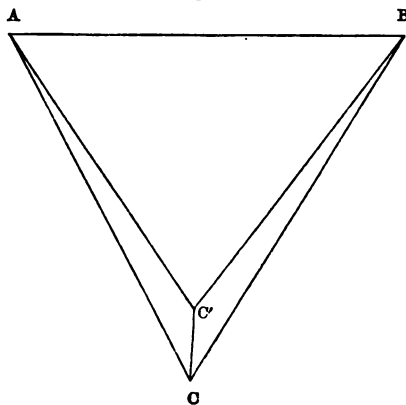
$$\therefore C \sim C' = A \sim B$$

which is the correction to be applied to the observed C' .

If C' is placed so as to fall within the angle ACB , as in figure 2, this correction becomes

$$C' - C = A + B.$$

Fig. 2.



In either case, we have,

$$\sin A = \frac{CC'}{CA} \cdot \sin CC'A,$$

and

$$\sin B = \frac{CC'}{CB} \cdot \sin CC'B,$$

whence A and B and therefore $A + B$ are known.

But unless the distances CA or CB are comparatively small, this correction is trifling, and generally inappreciable; and in an extended survey, where it is not only advisable but often imperative to have the principal stations very far apart, and a large fixed object may be necessary as a station mark, in order that it may be seen from

the required distance, the error arising from this is too small to be taken notice of, the difference of angle caused by a difference in position of a few feet practically vanishing when the object is at a distance of several miles.

When the stations are thus very far apart, considerable difficulty is often experienced in picking them up : a free use of whitewash is the ordinary way of making them more visible ; but there seems no reason why the plan of fixing a mirror so as to face towards the place of observation should not be more generally used by nautical surveyors. It has been used in the ordnance survey with very remarkable results, stations sixty miles distant having been readily observed by the aid of the sun glinting on the reflector. A large silvered ball fixed on top of a pole, at a suitable elevation, might perhaps be even better ; but I do not know that it has ever been tried. But where the requisite apparatus is available, nothing can compare with the limelight, which has been distinctly seen, even in daylight, at a distance of upwards of 100 miles.

When, as mentioned on a former page, it has been necessary or convenient to separate the end of the base line and the observing station, the triangulation can still, without any difficulty, be carried on from the base line, known only in *length*, until it reaches to the station astronomically determined : at this point the true meridian can be drawn in ; and, from the known latitude and longitude, those of every other point in the triangu-

lation are at once found, either by very ordinary trigonometrical computation, or by graduating the sheet from this known station, in the manner described in the next chapter.

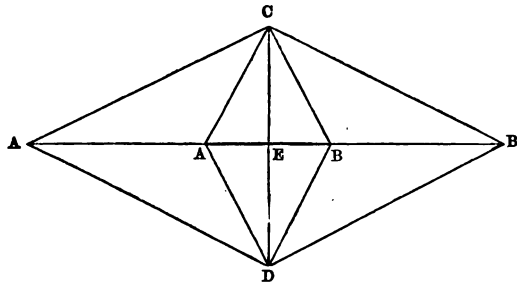
If, again, it has been convenient to commence the observations for the triangulation from this prime station, they can be carried on perfectly well till the end of a suitable base line is arrived at ; all the angles being taken in the careful and systematic manner already prescribed. For convenience of reference, these can be laid down in a free-hand drawing, the angular measurements being written in ; but clearly, as there is no known length, no exact plotting can be made. When the base has been measured, then, without further observations, the free-hand drawings can be interpreted and drawn in to scale, and the latitudes and longitudes determined exactly as before.

It has already been said that it is sometimes advisable to measure only a very short base line, and to calculate a longer one from other observations. If the instruments for taking angles be really good, this may be done very exactly in the following manner.

Let AB be the line actually measured. From E , the middle point, by estimation, of AB , pace EC and ED nearly equal to AB and at right angles to it, and set up flagstaffs C and D . The angles at A and B , as also those at C and D , should then be observed with the greatest possible nicety. The sides $AC, CB ; AD, DB$ can then be calculated ; and from the triangles ACD, BCD , the side CD can be found independently ; that is to say, each

triangle should be worked out separately, in order that one may be a check on the other ; the protraction, again, being a rough check on both. CD being thus determined, $A'B'$ may be laid off and calculated in exactly the same

Fig. 3.



way, and it is quite easy, if desirable, to pitch the poles A' and B' correctly in the line of AB ; $A'B'$ would then form a base line of about four times the length of the original one, AB . Of course, if a still longer line should be wanted, the multiplication can be carried on to any extent by repeating the construction.

Even when the whole line $A'B'$ is measured, in the manner already detailed, it is a useful precaution to note specially the length of a part of it, such as AB , near the middle ; to measure it again independently, with, if possible, even greater care, and from it to determine the whole length $A'B'$, by the construction and observations just described. In this way, any considerable mistake which may have been made in the measurement, such as the wrong counting of the pickets, would most probably

be detected, and might even be localised in its proper portion of the line, whether AA' or BB' , by the solution of the not very ill-conditioned triangles, ACA' , ADA' , BCB' , BDB' .

When the angles of the primary stations have been observed with the theodolite, they can be laid off at once on the chart; bearing in mind that each position is determined by the intersection of the lines already selected as sides of that particular triangle, and that the others are merely checks on its accuracy: when any discrepancy is noticed, it must of course be examined into and corrected; but, as a rule, if a complete round of angles has been taken at each station, there will be little difficulty in detecting the mistake. If the angles have been taken with the sextant, and the stations are not all on the same level, the observations will have to be corrected for the difference of altitude; this is easy enough, though tedious, the work of each being to find the angle at the zenith, that is, the difference of bearing, by the solution of the spherical triangle whose sides are the zenith distances of the two stations, and the angular distance between them. But even with the sextant, the horizontal angle may commonly be observed, with sufficient accuracy, by means of a plumb-line, in the manner already explained; or indeed, after some practice, without a plumb-line.

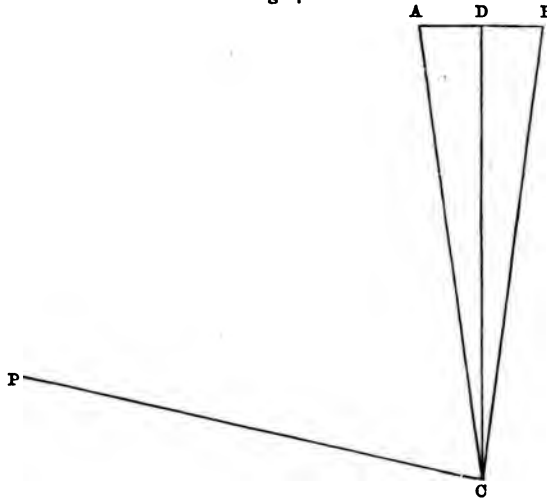
When these stations have been determined, from them any number of other positions can be fixed readily and accurately. These secondary stations, which do not enter at all into the main system of triangles, are chosen en-

tirely with reference to their topographical importance, and their position is laid down by such angles as can be best obtained, and, if necessary, by measuring the distance with the chain. In this work, the minute accuracy necessary for the determination of the primary stations is not required ; and triangles may be used which in the main triangulation would be quite inadmissible : the stations are marked entirely for themselves, and as no further observations connected with the triangulation are carried on from them, small errors in either distances or angles will not be repeated, and are of comparatively trifling consequence ; at the same time, all reasonable care should be taken, more especially in fixing such positions as, from their visible importance, are likely to be afterwards made use of as leading marks, such as the sharp peaks of hills, fantastically shaped rocks or trees, or the extreme points of tongues of land.

When as many secondary stations as convenient have been fixed in this way, the coast between them may be sketched in by hand, distances being determined by means of a 'ten-foot pole.' This is simply a pole of light wood, having near its ends transverse pieces, broad enough to be clearly visible, and exactly ten feet apart. An assistant carrying this pole walks on to a suitable distance, and there holds it, according to the observer's fancy, horizontal, or vertical, or inclined, but as nearly as possible perpendicular to the line drawn to the observer. The observer with his sextant then takes the angle between the cross-pieces, both on and off the arc, as the angle is small, the assistant meanwhile gently swaying the rod

backwards and forwards, till, in each observation, the largest possible reading is got on the sextant. Having this angle, the distance is easily calculated.

Fig. 4



Let AB represent the 'ten-foot pole,' and C the observer's position; ACB is then the angle observed, and the perpendicular distance CD bisects the angle ACB , and the line AB .

We have then

$$\frac{AD}{CD} = \tan \left(\frac{1}{2} ACB \right)$$

and

$$\frac{AB}{CD} = 2 \tan \left(\frac{1}{2} ACB \right).$$

But since the angle ACB is small,

$$2 \tan \left(\frac{1}{2} ACB \right) = \tan ACB \text{ very nearly,}$$

and we have

$$CD = 10 \cdot \cot CBA,$$

whence the distance may be at once found for any value of the angle ACB .

The distance which can be measured in this way necessarily depends on the distance at which the cross-bars can be clearly seen; but there is another point, namely this: as the distance increases, the angle diminishes, but more and more slowly as it gets smaller. If the angle is very small, a slight error in the observation will produce a much greater error in the distance than when the angle is not very small; and an error in the observation is more likely to be made when the distance is so great as to render the cross-bars at all hazy: an angle of $30'$ corresponds to a distance of 1,146 feet; and this is perhaps as far as the method ought, in a general way, to be trusted; but if the light is exceptionally good, shining full on the face of the cross-bars, and the observer is confident in the steadiness of his eye and hand, longer distances may occasionally be ventured on.

If the position C has been already fixed, as either a primary or secondary station, and the angle at C between D and some distant fixed position, such as P , is taken, the position D can be laid down with fair accuracy, in this way:—

Draw the line PC , and at the point C set off the angle PCD just observed; and mark off the distance CD , as determined. D is thus fixed; and as the observer moves on to D , similar observations will determine any number of consecutive points, whether along the coast line, or

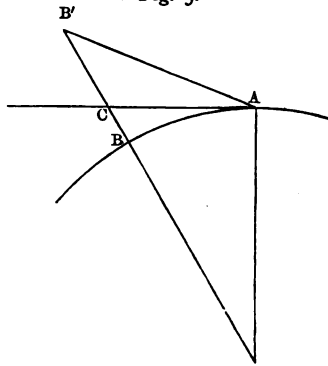
otherwise. In this way the outline of a coast may be laid down with very great accuracy, a continual check being afforded by breaking the work as each station already rigidly determined is arrived at.

The heights of all the neighbouring eminences which have any sufficient altitude are easily determined by altitudes taken from some two or three of the stations; the horizontal distance from these stations has been already found and laid down, so that the heights can be at once calculated from the plane right angled triangles; but when the distance is at all considerable, it becomes necessary to apply a correction for the curvature of the earth's surface.

The earth is so nearly a sphere, that, except in the largest and most exact trigonometrical surveys, no appreciable error is introduced by considering it as such; and on this hypothesis, points are said to be truly on the same level if they are at the same distance from the centre of the earth. But such points are not on the same *apparent* level. The difference is this: that an instrument adjusted by spirit level at any place has the axis of the telescope in the direction of a line which touches the sphere at that point, and the observed altitude of an elevated point is the angle between this line and the line drawn from the observer's eye to the object. Thus, if A B are two points on the surface of the earth, and B' a point vertically above B , the apparent altitude of B' observed at A is the angle $B'AC$. It must be borne in mind that in any figure showing this, the enormous

difference between the earth's radius, and any height, or any measured distance, cannot be represented, so that the figure is necessarily very much distorted ; thus, in reality, the angle $B'CA$ has scarcely any appreciable difference

Fig. 5.



from a right angle ; and no error worth noticing is introduced by considering it one, and so solving the triangle $B'CA$, to find $B'C$, the apparent height of the point B' ; but it is clear that to find the true height, the part BC must be added, and this part BC is the correction just spoken of. Its value is found in this way.

Let a denote the distance, measured or computed, from A to B , along the surface of the earth.

a' , the distance from A to C in a straight line.

r , the radius of the earth.

x , BC , the correction required.

Then, since DAC is a right-angled triangle, the square on

DC is equal to the sum of the squares on DA , AC ; that is

$$(x+r)^2=r^2+a'^2$$

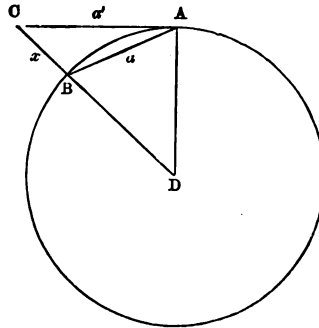
$\therefore$

$$x^2+2xr=a'^2$$

or

$$\frac{x^2}{2r}+x=\frac{a'^2}{2r}.$$

Fig. 6.



Now the difference between a and a' is so small, that no sensible error is introduced by writing a for a' ; also, x is such a very small quantity in comparison with r or $2r$, that the fraction $\frac{x^2}{2r}$ is practically inappreciable, and may be neglected, and our equation becomes simply

$$x=\frac{a^2}{2r}.$$

It is sometimes difficult for the untrained mind to understand the strict propriety of leaving out these algebraically small quantities; it is a point which so very much simplifies various computations, that it is worth our

while to dwell on it for a moment. Putting the case arithmetically, $2r$ is, in round numbers, about 7,000 miles; and within the limits of observation, x can scarcely be more than 2,000 feet, and is more probably less than 200 feet. The greatest value of the fraction $\frac{x^2}{2r}$ is thus

$$\frac{2000 \times 2000}{7000 \times 6000} \text{ or } \frac{1}{10} \text{th of a foot nearly ;}$$

whilst its more probable value is less than

$$\frac{200 \times 200}{7000 \times 6000} \text{ or } \frac{1}{1000} \text{th of a foot.}$$

It is scarcely worth further pointing out that not even the greatest of these values has any signification in determining the height of a mountain.

As the changes in the value of this correction, $\frac{a^2}{2r}$, depend entirely on the value of a , the distance of the object from the point of observation, it is easy enough to tabulate the different values it assumes for the different values of a , the earth's radius or r being at once deduced in nautical miles from the equation

$$2\pi r = \text{earth's circumference}$$

$$= 360 \times 60 \text{ miles}$$

$$\therefore 2r = \frac{360 \times 60}{3.1416} \text{ miles,}$$

whence if a and r are kept in miles, the value of the correction x , in feet, is

$$x = a^2 \cdot \frac{3.1416 \times 6075}{360 \times 60} \text{ feet.}$$

The logarithm of this numerical quantity increased by 10 is 9·946244, which is a constant log to be added to twice the log of a ; or, shortly,

$$\begin{aligned}\log x \text{ (in feet)} &= 2 \log a \text{ (in miles)} \\ &+ 9\cdot946244 \\ &- 10.\end{aligned}$$

But it may sometimes be more convenient to express a in feet ; in which case

$$2r = \frac{360 \times 60 \times 6075}{3\cdot1416} \text{ feet,}$$

and

$$x = a^2 \frac{3\cdot1416}{360 \times 60 \times 6075} \text{ feet,}$$

and the constant log to be added, as before, is 2·379152 ; the formula, in a logarithmic form, being

$$\begin{aligned}\log x \text{ (in feet)} &= 2 \log a \text{ (in feet)} \\ &+ 2\cdot379152 \\ &- 10.\end{aligned}$$

Unless a is large, this result will have a negative characteristic, and x will be very small ; up to a distance of 2,000 feet its value is less than $\frac{1}{10}$ th of a foot, and, except in very refined measurements, may be neglected ; it must be remembered, however, that it varies as the *square* of the distance, and that after this it increases rapidly. By means of the logarithmic formula just given, a table of its values may easily be formed, in this way :—

*Correction in Height for Curvature of Earth's Surface—
both in feet.*

Distance	Height	Distance	Height
1,000	·024	6,000	·862
2,000	·096	7,000	1·173
3,000	·215	8,000	1·532
4,000	·383	9,000	1·939
5,000	·598	10,000	2·394

It is, again, often convenient to apply the correction not as a length to the height, but as an angle to the observed altitude ; this angle is (fig. 6) BAC .

Then, this angle BAC is half of the angle BDC , so that if we denote the angular correction required by θ , ADB is 2θ ; and drawing DE perpendicular to AB ,

$$\sin \theta = \frac{a}{2r}$$

$\therefore$ in circular measure, since θ is very small,

$$\theta = \frac{a}{2r}$$

and

$$\theta'' = \frac{a}{2r \sin 1''}$$

Here again we have a constant quantity $\frac{1}{2r \sin 1''}$

whose logarithm increased by 10 is a constant, which being added to $\log a$, gives $\log \theta''$ at once. This constant log is 7·693577, where r and therefore also a is counted in feet ; and

$$\begin{aligned} \log \theta \text{ (in seconds of arc)} &= \log a \text{ (in feet)} \\ &+ 7·693577 \\ &- 10. \end{aligned}$$

If the distance is counted in miles, the radius must also be in miles, and the logarithm of 6075 must be added to the above ; and if the work is habitually done in miles, the constant log may be changed, by the addition of log 6075, to 11'477123, in which case we have

$$\begin{aligned} &\log \theta \text{ (in seconds of arc)} \\ &= \log a \text{ (in miles)} \\ &+ 11'477123. \end{aligned}$$

In addition to this correction for the earth's curvature, there is a correction for atmospheric refraction, which, at low altitudes, is sometimes very great. It is quite impossible to lay down any absolute value for this, for, in the lower strata of the atmosphere, the refraction is exceedingly irregular and capricious. Over desert plains, and more especially if these plains are salt, it seems to be always so, giving rise to the remarkable appearances known as mirages ; but instances of unequal refraction and extraordinary mirage are frequent in the Arctic, and high latitudes, even over the sea ; in temperate and tropical latitudes, mirage over the sea is not very common. One decided rule may, however, be laid down, that when the altitudes are low, they should never be observed when there is reason to suspect any peculiar refraction, which may often be detected by the distorted appearance of familiar objects, trees, houses, rocks, &c., and, more closely, by the altitude of some hill observed always from exactly the same place. Irregular refraction is met with more in the heat of the afternoon than in the forenoon ; and may always be suspected when the sun is shining

fiercely over dry ground ; but even in the cool of the evening very curious effects have been observed.

The propriety of observing at all, on any special occasion, must thus be judged by the surveyor ; and the correction to be applied is entirely an empirical one, determined by the average of many observations, in which the refraction, under ordinary circumstances, has been carefully measured either by means of the dip sector, or by reciprocal angles,—an extension of the subject into which it is unnecessary here to enter.

This average enables us to lay down a rule which seems to hold with an approximation sufficiently correct for ordinary determinations of height. It is this :—

From the distance of the observed object, subtract $\frac{2}{3}$ ths of itself, and with this reduced distance find the correction for curvature, as already laid down ; the result is the combined correction for *both* curvature and refraction ; but to find the height, the true distance is used.

This rule may be modified thus :—

Subtract $\frac{2}{3}$ ths of the distance in miles, considered as minutes of arc, from the altitude, and work throughout with the altitude so reduced.

The rule is, however, not so exact as to make it necessary rigidly to use such an inconvenient fraction as $\frac{2}{3}$; $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$, according as the division is simplest, is, in a general way, near enough.

As has been said, this correction for refraction is not exact ; and when strict accuracy is required, it must be avoided. But in this case, the observations are through-

out of a more rigorous and detailed nature ; and the height of any eminence, or the difference between the true level of any two stations, is actually measured by a process known as 'levelling.'

The instrument called a 'level,' with all its refined adjustments, is, in its simple form, a telescope which can be laid perfectly horizontal by means of a spirit level attached. The axis of the telescope, shown by cross wires, is thus a tangent-line to the earth's surface at the point where it is set up ; so that if a staff on which the divisions to the $\frac{1}{100}$ th part of a foot are clearly painted in black and white, is held upright by an assistant, in the direction to be levelled, the observer, looking through the telescope, reads on the staff the feet, tenths, hundredths, opposite the cross wires ; and—leaving for a moment the corrections for curvature and refraction on one side—the difference between this reading and the height of the axis of the telescope is the difference in the level of the two places.

The staff must be held perfectly upright. A practised assistant will hold it nearly so without difficulty ; and the exact position of perpendicularity is determined by gently swaying the staff backwards or forwards, to one side or the other, till the observer is satisfied that he has got the *least* reading.

The foot of the staff must rest on the ground, but must not be allowed to sink into it. If the ground is soft, a piece of slate, or sheet iron commonly supplied for the purpose, is laid for the staff to rest on ; or some observers prefer driving in a wooden picket flush with the surface

of the ground, on which the foot of the staff rests during the observation, and which remains to mark the station if the work has to be repeated or verified.

But in a single observation, such as I have just described, the curvature and refraction would have to be corrected for. To avoid the inaccuracy which would thus necessarily enter into the work, the observation is divided into two; that is to say, the level is placed midway between the two stations, and a staff, held upright at each of them, one forward and one back, is observed. The difference between the readings on the two staves is the true difference of level between the two stations, free from all errors on the score of curvature or refraction; for the level being midway between the staves—and when extreme accuracy is required the distance should be chained—the corrections for curvature and refraction are exactly the same for both, and do not affect the difference of the observed readings.

Thus, if h , h' be the two readings, x the correction for curvature, and y the correction for refraction, we have, as the difference of level,

$$(h+x-y) \sim (h'+x-y) = h \sim h'.$$

The distance between the two staves is practically limited by the power of the telescope and the capability of the observer to read off the divisions; but as a general rule it should not exceed 500 feet, for even if the readings were clear, greater distances, especially if the ground is hot, might admit some accidental vagary of the refraction. When, therefore, as is commonly the case, much longer

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distances have to be levelled, the observation has to be carried forward by a succession of staves, read in the same manner, and with the same attention of detail ; the difference of level between the first and last station is then at once obtained, being the difference between the sum of all the *fore* readings and the sum of all the back readings, the position left or the position arrived at being the highest, according as the sum of all the fore readings, or the sum of all the back readings is the greatest. As a check on the arithmetical part of the work, it is well to find each difference separately, arranging them in columns under distinct heads according as the fore or back reading is the greatest ; the final difference of the sum of these differences should of course be the same as the other. The sights, as taken, must be entered at once in the note book ; and the little work attending the reduction may most conveniently be tabulated, in this way :—

<i>(Date)</i>		<i>(Place)</i>	<i>(Observer)</i>		<i>(Instrument)</i>
Back	Fore	Rise	Fall	Remarks, &c.	
2'38	3'55		1'17		
5'21	4'15	1'06			
4'38	3'99	0'39			
4'38	3'39	0'99			
16'35	15'08	2'44	1'17		
15'08		1'17			
1'27		1'27			

that is, the place arrived at is 1'27 feet above the place left. And in this manner the level can be carried over the highest accessible mountain tops, or over the widest possible extent of country.

What I have here said is a very bare outline of the work ; the laying the level truly and quickly, as well as the reading off of the staves, requires both attention and practice, and it is only by attention and practice that it can be learned.

It would be out of place to discuss here the methods of determining heights by observations of the barometer, or of the boiling point of water ; such determinations do not properly fall within the duties of a nautical surveyor, whose elevations can always be referred at once to the sea level, either by actual levelling or by trigonometrical work, such as we have been speaking of. The barometric and boiling point methods, valuable as they undoubtedly are, are suitable rather for the traveller and explorer of unknown countries than for the accurate surveyor ; but if, in the course of a survey, it should be considered desirable to make use of them, they are very fully explained in many different and well-known works, some one or other of which is sure to be available.

CHAPTER III.

CONSTRUCTION OF CHARTS.

The different Projections of the Sphere—The Gnomonic Projection—
Methods of Graduating Sheets—Figure of the Earth—Amount of
Finish requisite.

WHEN we attempt to represent any portion of the earth's surface on paper, we are at once met by the difficulty caused by its curvature ; it is absolutely impossible to project the curved surface on a plane without more or less distortion, which, when the extent of country is large, becomes so considerable as completely to destroy the true proportion between the parts.

The orthographic projection, which is a right projection on the plane of any great circle, shows the central regions with a very close approximation to accuracy ; but when it is applied to a whole hemisphere, the countries near the margin appear folded up, as it were, one on top of another, in a confused jumble. The stereographic projection, which is a projection on the plane of any great circle by lines drawn from one of the poles of that circle to the several points in the opposite hemisphere, gives a representation of that hemisphere,

which, in its general appearance, is much less distorted than the other ; but the distortion is still considerable, and renders it quite unfit for purposes of accurate delineation.

Although these two, as the natural projections of the sphere on a plane, the one from a point infinitely distant, the other from the pole of the plane, are always spoken of in geodesical treatises, they have quite fallen out of any practical application ; maps of complete hemispheres being shown on what is called the equidistant projection, which is, strictly speaking, not a projection, but merely an artificial development.

Describe a circle as the meridian on the plane of which the hemisphere is to be projected, and draw two diameters at right angles to each other, one as a primary meridian, the other as the equator.¹ Divide these radii and each quadrant severally into nine equal parts, as spaces of 10° . Then draw the meridians as arcs of circles passing through both poles and the successive divisions on the equator, and draw the parallels of latitude as arcs of circles passing through the corresponding divisions on the two primary meridians, the one shown by the bounding circle, the other by the vertical diameter. The map being thus divided into spaces of 10 degrees in latitude and longitude, the position of as many points as possible is marked in as correctly as the scale permits ; and the outline is finished off in the usual manner. The maps so drawn give a good general idea of the configuration

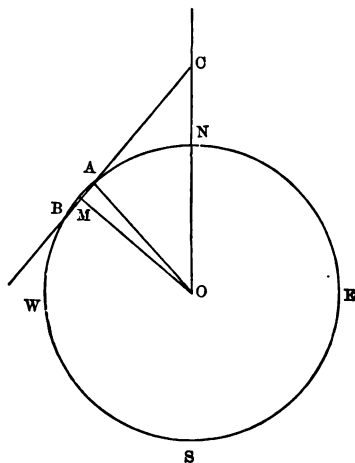
¹ It is unnecessary here to refer to a particular figure ; any ordinary map of the 'World in Hemispheres' will be sufficient.

of countries, and of the distribution of land and water, but are of no use for any purposes of accurate measurement.

The projection used by map makers for large extents of country—as, for instance, Europe, or any of its kingdoms—is known as the conical projection, and is drawn as if projected from the centre of the earth on the surface of a cone cutting the surface of the earth in the parallels of latitude equidistant from the extremes and middle of the required limits. It is constructed in this way.

Suppose the map is to embrace country between the 40th and 60th parallels.

Fig. 7.



Draw a vertical line as a primary meridian ; and, according to the scale desired, set off on it spaces repre-

sending the degrees of latitude from 60° to 40° . Through these draw the parallels of latitude as arcs of circles, the centre being a point in the primary meridian produced, supposed to be the vertex of the intersecting cone.

Let the circle *NESW* represent any meridian, of radius *r*, and let *CAB* be a generating line of the cone cutting the circle in the points *A* and *B*, which in the case we are considering are in latitude 55° and 45° respectively. Then *OM*, the perpendicular drawn from *O*, bisects *AB*, and the point *M* represents the latitude of 50° .

Draw *OA*, and we have

$$\begin{aligned} OM &= r \cdot \cos 5^\circ \\ \text{and } CM &= OM \cdot \cot 50^\circ \\ \therefore CM &= r \cdot \cos 5^\circ \cdot \cot 50^\circ, \end{aligned}$$

which, taking *r*, expressed in lengths of degrees, as $\frac{180^\circ}{3 \cdot 1416}$, gives

$$CM = 47 \cdot 894 \text{ degrees.}$$

Measuring, therefore, on our primary meridian, from the central point, a distance equal to $47 \cdot 894$ degrees, according to the scale used, determines the point which is the centre of the arcs described as parallels of latitude.

On the arcs which represent the parallels of 45° and 55° , in which the cone intersects the sphere, set off degrees of longitude, of the length proper to the latitude, measuring them each way from the primary meridian; straight lines, drawn through the corresponding degrees of longitude on these two parallels, are the meridians; these clearly converge to the centre of the parallels of

latitude, which, therefore, if not too distant, may be used as a check on the correctness of the work. As in the former case, the map is then finished off by marking in the places, according to their latitude and longitude, and filling in the outline.

Within moderate limits, the distortion attending this projection is quite inappreciable, and even for a considerable extent of country it is but trifling; on this account, it, or some slight modification of it, is now generally used by all our best map makers.

For purposes of accurate measurement it is, however, subject to the disadvantage that, with the exception of the meridians, all great circles are represented on it by curved lines. Now all measurements between places on the earth's surface are necessarily made on the arc of a great circle, and for these to be accurately represented on paper it is of the utmost consequence that the projection of every arc of a great circle should be a straight line.

It is this necessity which prevents our using the orthographic projection already spoken of, which, though subject to extreme distortion near the margin of the hemisphere, is, near the centre, the most accurate of all the projections, as is at once seen if we suppose the primitive plane—the plane on which the projection is made—to be, not that of a great circle, but a tangential plane parallel to it.

It is the same necessity which, more than its extreme distortion, prevents our using the projection known as 'Mercator's;' for on it also all great circles, except the meridians and the equator, are represented by curved

lines. The construction of charts on Mercator's projection, and the peculiar and important advantages which it offers to the navigator, are so thoroughly described in the ordinary text books of navigation, that it is needless to say anything more about them here ; the more so, as the projection cannot be made use of for the accurate work of the surveyor.

For this work we adopt what is known as the gnomonic projection, which is formed by lines drawn through the several points from the centre of the sphere to a plane touching the sphere in a point near the middle of the country to be represented. This projection, it will be at once seen, fulfils the required condition of showing every great circle as a straight line ; and though the distortion to which it is subject is greater than that of the orthographic or even of the conical projection, it is, within limits, which, though small, are a good deal beyond those which a surveyor's chart can embrace, so trifling as to be practically inappreciable. At a distance of 60 miles from the central point, a mile, so projected, is about *one foot* too long ; and no one chart on a moderately large scale can possibly extend over 120 miles. The paper is not made large enough ; and, if it was, the chart cabin of a surveying ship could not admit it. For detailed plans, drawn on the largest scales, six, eight, or twelve inches to a mile, and which are therefore necessarily of very limited extent, the projection is more accurate than the finest drawing instruments ; and as the scale diminishes, and the extent of country increases, the distortion is equally inappreciable : at a distance corresponding to

four degrees of latitude from the central point, the mile is increased by about 25 feet or $\frac{1}{2560}$ th of its length ; the scale which will show the 1-250th part of a mile is much larger than can be used for any chart extending over eight degrees of latitude. For any detailed charts such as are drawn by the surveyor in the progress of the survey, the gnomonic projection is therefore, practically speaking, perfectly accurate ; and having, alone of all the projections of the sphere, the property of representing *all* great circles by straight lines, cutting each other at angles, which, within the required limits have no sensible difference from the angles at which the circles cut each other, it, or some slight and simple modification of it, is invariably used by surveyors. It is therefore necessary for us to examine, in full detail, into the method of constructing it.

The meridians, being great circles, will of course be represented by straight lines ; but the parallels of latitude, which are small circles, will appear in the projection as curve lines, which, it is easily shown, are some one or other of the conic sections ; being, in fact, the intersections, with the primitive plane, of the cone whose vertex is the earth's centre, and whose directrix is the parallel of latitude under consideration. According as the latitude is greater than, equal to, or less than 45° , the curve will be an ellipse, a parabola, or an hyperbola ; the projection of the parallel of middle latitude having in each case its vertex at the central point—that is to say, the point in which the plane touches the sphere—and its axis along the meridian ; but whichever of the

conic sections it may happen to be, the very small portion of it which has to be described does not sensibly differ from the circle of curvature at its vertex, and may therefore be described as a circle whose radius is the radius of curvature.

The investigations necessary to determine the value of this radius of curvature require a knowledge of mathematics somewhat more extended than I suppose to be possessed by those of my readers for whom I principally write; others who wish to examine more thoroughly into the subject will find the general propositions for determining the axes of the sections in Hymers' 'Astronomy,'¹ from which it follows that

$$\rho = r \cot l$$

where ρ is the radius of curvature required,
 r the radius of the earth
 and l the latitude.

Suppose, for instance, that the central point of the chart is in latitude 35° . To find the radius with which the parallel of latitude is to be described, we have

$$\begin{array}{r} \log \cot 35^\circ \quad \quad \quad .154773 \\ \log \left(r = \frac{180 \times 60}{3.1416} \right) \quad 3.536273 \\ \hline \quad \quad \quad 3.691046 \end{array}$$

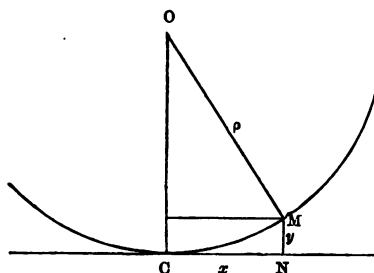
$$\rho = 4909.6 \text{ miles.}$$

It is at once evident that to describe a circle with such a radius, on any useful scale, is a practical impossibility;

¹ Second edit., App. I. Arts. 13, 14.

but it is no very difficult matter to calculate the distances of any number of points in the circle from the line drawn through the central point touching the circle, that is to say, at right angles to the meridian ; and determining, in this way, a number of points in the curve—as many and as close together as may be thought necessary—the curve is drawn simply as a series of straight lines joining these points.

Fig. 8.



Let C denote the central point, CO the meridian through C , CN the tangent to the circle at C , and therefore perpendicular to CO . Along CN set off any distance $CN=x$; and from N draw NM perpendicular to CN , meeting the circle in M : let $MN=y$. Draw OM . Then, denoting the radius, OC or OM by ρ we have

$$\begin{aligned}\rho^2 &= (\rho - y)^2 + x^2 \\ (\rho - y)^2 &= \rho^2 - x^2 \\ &= (\rho + x) \cdot (\rho - x)\end{aligned}$$

$$\text{and } \log (\rho - y) = \frac{1}{2} \{ \log (\rho + x) + \log (\rho - x) \},$$

from which y is easily found for as many different values

of x as we choose. Thus, for instance, in latitude 35° , taking the minute of the meridian as our unit, we have $\rho=4909\cdot6$; and assuming x as successively 5, 10, &c., we get the corresponding values of y , which we form into a table like this :

x	y	x	y
5	'00	35	'13
10	'008	40	'17
15	'02	45	'21
20	'03	50	'25
25	'06	55	'31
30	'09	60	'38

which may be carried on as far as necessary.

Having formed this table, we are ready to begin the construction of the chart.

Through the middle point of the paper, and parallel to its edges, rule two straight lines at right angles to each other, one the meridian, the other the tangent. Along the meridian set off the minutes of latitude, according to the scale chosen, counting each way from 35° ; and along the tangent set off, on the same scale, the several lengths of x given in the table just formed, and at each division draw a short perpendicular and set off the corresponding value of y , as given in the table, always according to the same scale. The upper point of each of these ordinates so set off is a point in the curve representing the parallel of latitude, which may then be drawn by joining all these upper points by straight lines. Other parallels of latitude may be drawn in the same way, by drawing through the

chosen latitude, as shown on the primary meridian, a straight line, perpendicular to the meridian, setting off on it the values of x , and raising from these points ordinates equal to the corresponding values of y , and joining the upper points of these ordinates exactly as before; this will draw the parallels of latitude as arcs of equal circles, which theoretically is not quite correct; but the actual difference in the lengths of the ordinates of the different circles is extremely small in comparison with their length, that is, in comparison with the values of y ; and these values are themselves extremely small, as we see from the table, so that no appreciable error is introduced by considering the circles as equal; our drawing instruments, in fact, are not capable of showing them unequal within the limits of the chart.

The parallels of latitude being thus drawn, along the central one, as also along the upper and lower, set off the minutes of longitude, of a length having the proper ratio to that of the minutes of latitude, according to the scale used. Thus, in latitude 35° , a minute of the meridian is 6066.23 feet, and a minute of longitude is 4991.94 feet; these numbers are to each other in the ratio 1 : .82; whatever, therefore, our scale may be, the length of a minute of longitude is .82 of the length of a minute of latitude; if the scale is 1 inch to the mile, the minute of longitude is .82 of an inch; if the scale is 6 inches to the mile, the minute of longitude is 4.82 inches. Similarly, the minutes of longitude are set off on the upper and lower parallels of latitude, each to their proper length;

and the meridians are drawn as straight lines, passing through the corresponding divisions on the parallels of latitude, and at such intervals as, from the scale used, may seem desirable ; if the scale is very large, a meridian may be drawn at every minute or every half minute ; if not so large, every two minutes, or every five minutes, or ten minutes, may be sufficient.

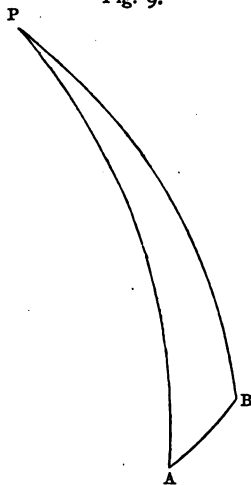
The graduation of the chart is then complete, and the work of the survey may be drawn in.

The method of graduating a chart which I have just explained is both accurate and simple ; the chief difficulty about it is the construction of the table of ordinates, for laying off the parallels of latitude, and this is really no very formidable undertaking ; but, probably because the method has been nowhere clearly explained, it has not been much used, and surveyors have commonly graduated their charts by another method which is perhaps not less accurate, and in which the preliminary calculations, although quite as long, are of a more familiar nature.

Choose two points of convenient latitude and longitude, one near the south-western margin of the proposed chart, the other near the north-eastern ; thus, if we suppose our central point to be in latitude 35° N., and longitude 10° E., and that the chart is to extend over 4 degrees of latitude and longitude, we may conveniently choose a point *A*, in lat. 33° , long. 8° ; and another *B*, in lat. 37° , long. 12° . The length of *AB* is then calculated from the spherical triangle *APB*, where *P* is the pole, in which we have

given the two colatitudes AP , BP , and the difference of longitude, the angle APB .

Fig. 9.



From this we get

$$AB = 5^{\circ} 10' 9''$$

$$= 310.15 \text{ miles}$$

$$\angle PAB = 38^{\circ} 11' 30''$$

$$\angle PBA = 139^{\circ} 31' 30''$$

and its supplement, $40^{\circ} 28' 30''$.

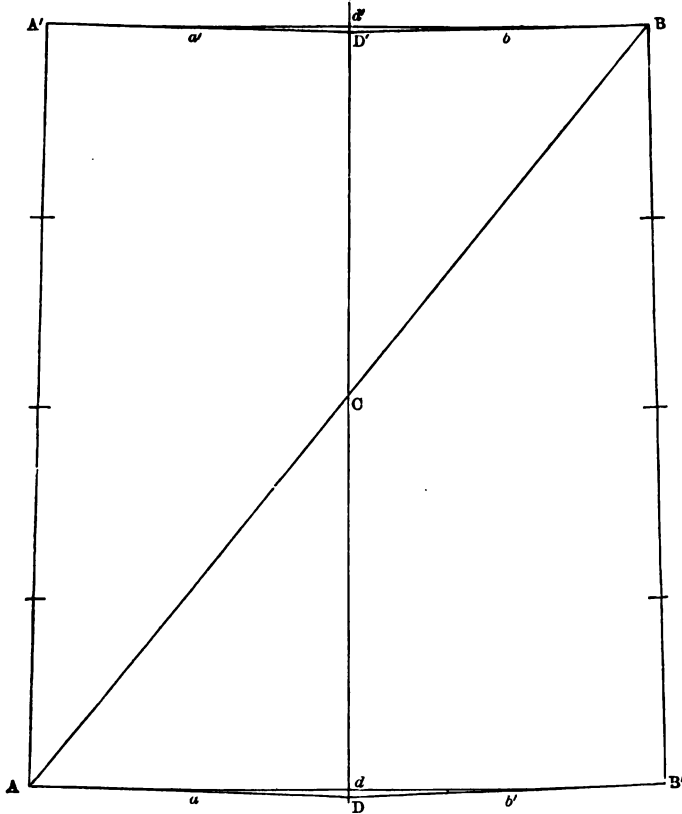
Then, in order that the chart may lie symmetrically on the paper,¹ find the mean of these two angles, PAB , and the supplement of PBA , which, it may be pointed out, are the inclinations of AB to the meridians of A and B ; or, in other words, are the true bearings of A and B

one from the other. This mean is $39^{\circ} 50'$. Lay the straight edge vertically, near the left side of the paper, and at a point in it near the bottom, set off with a protractor the mean just found, $39^{\circ} 50'$; draw the line which makes this angle upwards towards the right-hand top corner, and lay off on it the length AB , representing, according to the required scale, 310.15 miles, in such a way that the

¹ 'Nothing betrays want of forethought more than the meridian being out of the parallels to the sides of the paper, the work hedged up into one corner, or pieces pasted on when plenty of blank paper remains on the original sheet, not perhaps even devoted to soundings.'—BELCHER.

middle point of AB may be near the middle of the paper.

Fig. 10.



At the points A and B we then set off the angles $BAA' = 38^\circ 11' 30''$, and $ABB' = 40^\circ 28' 30''$.

These lines AA' , BB' , are the meridians through the points A and B . On them set off AA' , BB' , each equal to the difference of latitude, 4 degrees; so that A and B' , B and A' , are respectively on the same parallel of latitude.

Find the points midway between A and B' , A' and B . In doing this, do not complicate the construction by drawing the lines AB' or $A'B$; a small piece, say about an inch of each, is quite sufficient, as the middle point can be estimated to within some such limit. Then, by joining these points, draw the line DD' , which is the meridian of middle longitude.

From A and A' draw lines perpendicular to AA' , and from B and B' draw lines perpendicular to BB' ; the corresponding perpendiculars will meet in points in the mid-meridian DD' .

Let D and D' be the points in which they meet; then, $AD=B'D$, and $A'D'=BD'$.

Bisect each of the lines AD , $B'D$; $A'D'$, BD' , in the points a , b' ; a' , b ; and join ab' , $a'b$, cutting the meridian DD' in the points d , d' .

Again bisect Aa , ad , db' , $b'B'$; $A'a'$, $a'd'$, $d'b$, bB ; and join every two adjacent points of the bisection.

Repeat this process of bisecting the lines and joining the adjacent points of bisection, and continue repeating it until the broken lines AdB' , $A'd'B$, do not appreciably differ from continuous curves. These curves are the parallels of latitude.

On these set off the degrees and minutes of longitude in the manner already explained, and rule as many meridians as may be wanted. And as many parallels of

latitude as may be wanted are drawn in, either by going through the same process as before, or, which is simpler, by setting off the required latitude on the several meridians and joining the points of equal latitude. The graduation is then finished off to any required detail, and the survey is worked in as before.

The point C in which the meridian DD' meets the line AB does not bisect AB , and though its distance from the point of bisection is probably trifling, it may be considerable if the limits of the chart are at all large, as in the particular case I have just described. But if the chart is on a large scale, and the extent of country to be plotted necessarily very small, C may be taken as the point of bisection of AB , without any appreciable error.

In this case we would begin by drawing the meridian DD' through C , a point chosen near the middle of the paper; would draw the line ACB cutting DD' at an angle equal to the mean of the two true bearings; and would set off the distances CA , CB , each equal to half the whole distance AB . The meridians AA' , BB' , are then drawn, and the distances AA' , BB' set off, exactly as before. We can also set off Cd , Cd' each equal to half the difference of latitude; and through the points A , A' ; d , d' ; B , B' ; draw lines respectively perpendicular to the three meridians, the corresponding perpendiculars meeting in the points a , b' ; a' , b ; then go on exactly as before, bisecting and joining till the broken lines become continuous curves.

In low latitudes, and, as in the case we have just been speaking of, in charts of small extent, the meridians, and

the parallels of latitude do not diverge appreciably from parallel straight lines, and may be drawn as such ; in high latitudes the divergence is greater. The necessity for taking it into account is at once shown by a comparison of the two true bearings ; if these are so nearly the same that it is impossible, with the instruments used, to protract the difference, it evidently cannot be drawn, and the meridians are sensibly parallel.

To avoid complication, I have described these methods of graduating a sheet in the simplest form, without referring to the draught of the survey which the sheet has to contain. But in practice experienced surveyors commonly prefer working from points actually observed, and plot the work of the survey on the sheet blank, graduating it afterwards. If the work was perfectly correct, the plotting and the graduation, independently put in, would of course accurately agree. But as a matter of fact, work never is, and cannot be perfectly correct ; eye and hand do not admit of it ; and in a large sheet a point may not come to its proper latitude and longitude by perhaps half an inch. Supposing of course that no evident mistake has been made, this is due to an extremely slight error in the scale, which, being multiplied several times in the length and breadth of the sheet, may at last amount to something very noticeable. It would be never-ending work to go over the whole again, probably only to find it as much out in the opposite direction ; and it is more satisfactory to graduate the chart to the draught, and strictly according to the same scale, which may be the hundredth part of an



inch too long or too short. In this case, the graduation has to be commenced from two points *A* and *B*, actually plotted, chosen as before, near the opposite corners of the chart, but whose latitudes and longitudes, known by trigonometrical computation, are not of even minutes. With the exception of the necessary reductions, the work does not differ from that already described.

When the meridians through the points *A* and *B* have been drawn, set off along these from *A* and *B*, and towards the south or north respectively, the fractional parts which the latitudes want of the nearest *outer* minute of latitude ; at the points so set off draw short perpendiculars outwards, and set off on these the fractional parts which the longitudes want of the nearest outer minute of longitude. The points so fixed are the extreme corners to the S.W. and N.E. ; the outer meridians can be drawn parallel to those already drawn through *A* and *B*, for at such a small distance the divergence is inappreciable ; along these meridians the minutes of latitude are set off, and the parallels of latitude are drawn in exactly as before.

In concluding the description of these methods of graduating sheets, it is perhaps as well to say that the figure, page 89, to which I have referred, is, in order to show at all the convergence of the meridians and the curvature of the parallels, necessarily drawn on a scale quite unknown in surveying. The smallest scale ever used for plotting original work is perhaps 1 inch to a mile ; 4 or 6 inches to a mile is commonly used for accurate and detailed surveys of some extent ; for harbour work, a scale of 18, 24, or 30 inches is sometimes used ; and

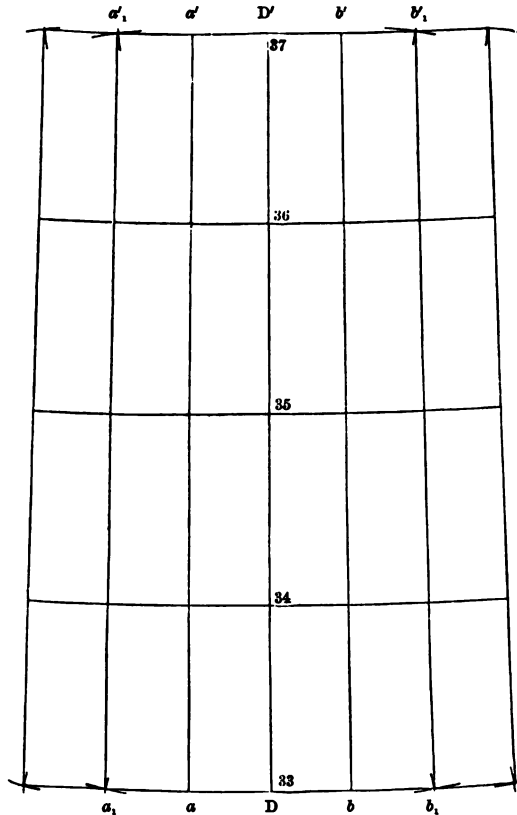
some of the surveys about our own coast have been laid down on a scale of 146 inches to the mile.

The methods of graduating a chart which I have just described aim at the greatest attainable accuracy, and should be used when the survey is of a detailed and rigorous character ; but hasty surveys are occasionally carried along considerable extents of coast line, though not so frequently now as in former days, in which extreme accuracy is not aimed at, or rather is impossible, from the rude nature of the observations, which alone the circumstances permit. A survey taken in this way is properly known as 'a running survey,' and the methods of conducting it will be described in a future chapter; my object in alluding to it here is merely to point out that, when the observations neither are nor can be more than approximations of no very correct nature, to graduate a chart, on which to lay them down, with all the nicety of measurement which we have been considering, would be merely waste of time. For work of this kind a somewhat simpler process has been devised, which is quite sufficiently accurate for the purpose, and is indeed more accurate than it pretends to be. Being used generally for coast lines of some extent, it is commonly drawn on a smaller scale than other more exact charts, but over 4 or 6 degrees of latitude it answers perfectly well.

Through the middle point of the paper draw DD' as a primary meridian, and lay off on this, according to the scale chosen, the degrees of latitude through which the chart is to extend. Let us suppose, as before, the limits

of our chart to be from 33° to 37° N., and from 8° to 12° E.

Fig. 11.



Through D and D' , the points of extreme latitude, draw short perpendiculars to the meridian DD' , and on

these set off Da, Db ; $D'a', D'b'$, respectively equal to half the length, according to scale, of a degree of longitude in those latitudes.

Now in lat. 33° a degree of longitude is equal to 50.562 minutes of the meridian, and in lat. 37° is equal to 48.127 ; hence we have the proportions,

$$60 : \frac{1}{2} \text{ of } 50.562 :: 1 : .421$$

and $60 : \frac{1}{2} \text{ of } 48.127 :: 1 : .401$

and set off Da, Db , each equal to $.421$ of the length of a degree of latitude, from 33° to 34° ; and $D'a', D'b'$, each equal to $.401$ of the degree of latitude, from 36° to 37° .

Set the distances $a'b$ or $b'a$ with the beam compasses; and with this as a radius, and D' as a centre, sweep out the little arcs, a_1, b_1 ; and again, keeping the same radius, and with D as a centre, sweep out the little arcs a'_1, b'_1 . Then with centres a and b, a' and b' ; and distances respectively aD or $bD, a'D'$ or $b'D'$; describe little arcs, cutting those just drawn in the several points $a_1, b_1; a'_1, b'_1$.

Draw the lines $aa', bb', a_1a'_1, b_1b'_1$, which represent the meridians for each half degree of longitude, through two degrees.

Draw also the lines $aa_1, bb_1, a'a'_1, b'b'_1$; and the broken lines $a_1aDbb_1, a'_1a'D'b'b'_1$, represent the parallels of latitude through this extent.

The same process is then repeated, the long and the two short radii remaining constant throughout, the centres being successively $a', b'; a, b$; and $a'_1, b'_1; a_1, b_1$, which will bring the chart to the proposed limit; but it may be

continued in exactly the same way through any extent thought desirable, which ought not perhaps to exceed 5 degrees each way from the primary meridian.

The degrees of latitude are then set off on the several meridians, as *aa'*, *bb'*, &c. ; and the parallels of latitude are drawn as broken lines joining the points of equal latitude.

A certain increased pretension to accuracy might be given by working to quarter degrees of longitude, instead of half degrees, in the manner described, but the difference would be very slight, and the advantage gained would not compensate for the additional labour.

In discussing the different methods of graduating charts, I have spoken throughout of the earth as a sphere. The divergence from the spherical form over any small extent of surface, is, in fact, quite inappreciable ; and it is only in the large trigonometrical surveys that it becomes necessary to consider the really spheroidal shape. With that we have no concern here ; the only way in which it affects a nautical survey, or any survey embracing but a small extent of country, is the difference which it causes in the lengths of minutes of the meridian in different latitudes. The geographical mile is generally defined to be the length of a minute in arc of the earth's equator ; but the nautical mile, as defined by hydrographers, is the length of a minute of the meridian, and is different for every different latitude ; it is equal to a minute in arc of the circle whose radius is the radius of curvature of the

meridian at the given point, which is determined by the expression,¹

$$\rho = a - 2c + 3c \sin^2 l$$

where ρ is the radius of curvature required

a the equatorial radius of the earth

$$= 3962.8 \text{ statute miles}$$

c the difference between the equatorial and polar radii

$$= 13.24 \text{ statute miles}$$

l the latitude ;

and ρ being known,

$$\text{minute of the meridian (in feet)} = \rho \sin 1'.5280.$$

Suppose, for instance, that we have to find the length of a nautical mile in latitude 35° ,

$$c = 13.24$$

$$3c = 39.72$$

$$\text{Log sin } 35^\circ \quad 9.758591$$

$$\text{Multiply by 2} \quad 9.517182$$

$$\text{log } (3c) \quad 1.599009$$

$$\hline 1.116191$$

$$\text{Nat. number} \quad 13.06$$

$$a \quad 3962.82$$

$$(a + 3c \sin^2 l) \quad 3975.88$$

$$2c \quad 26.48$$

$$\rho = 3949.40 \text{ statute miles.}$$

¹ Hymers' 'Astronomy,' Art. 125.

log ρ	3.596531
log 5280	3.722634
Log sin 1'	6.463726
	3.782891

Nat. number 6065.8 feet,

the length of the minute of the meridian in latitude 35° .

The minutes of longitude corresponding to the different latitudes are also affected by the spheroidal shape of the earth, and are somewhat longer than their values as calculated on the sphere by the ordinary formula,

$$\text{min. of long.} = \text{min. of lat.} \times \cos \text{lat.}$$

They are more correctly found by the formula,¹

$$\begin{aligned} &\text{minute of longitude (in feet)} \\ &= \cos l (a + c \sin 2l) \sin 1'.5280. \end{aligned}$$

Thus, to find the length of a minute of longitude in latitude 35° ,

Log sin 2l	9.517182
log c	1.121888
	.639070
Nat. number	4.36
a	3962.82
(a + c sin 2l)	3967.18

¹ Hymers' 'Astronomy,' Art. 126.

its log	3.598462
Log cos 35°	9.913364
Log sin 1'	6.463726
log 5280	3.722634
	3.698186

Nat. number . . . 4991 feet,

which is the length of a minute of longitude in latitude 35°.

A very convenient set of tables, giving the length in feet of minutes of latitude and longitude for every 10' of latitude, has been published by Mr. Carrington, of the Hydrographic Department,¹ and if at hand will save the trouble of making the calculations which I have just explained; otherwise, the work will have to be done; and, after all, the numerical computations are not very formidable, though the investigations on which they are founded require a knowledge of conic sections.

With this exception, the spheroidal shape of the earth does not affect a nautical survey, and any further consideration of it would be out of place here. In the reduction of observations, the earth is considered to be simply a sphere; and when the positions observed are at any great distance from each other, the triangles with which we have to do cannot, with perfect accuracy, be considered as plane triangles; they are, in fact, spherical triangles, but with this peculiarity, that the radius of the

¹ Potter, 1868.

sphere on which they are described is enormously large in proportion to the sides of the triangles.

The various methods by which these triangles are reduced are fully discussed in any standard treatise on spherical trigonometry; and their skilful and correct application forms a very important part of the work of a great trigonometrical survey. Into a nautical survey, unless of the most extended nature, such problems but rarely enter, and on these rare occasions sufficient accuracy is obtained by the simple rule :—

Subtract from each *horizontal* angle (whether observed or reduced) one-third of the spherical excess, that is, of the quantity by which the sum of the three angles exceeds 180° . Then solve the triangle as a plane triangle; and the sides so determined are the distances between the stations.

In describing the several methods of graduating a chart and plotting the survey, I have supposed the reader familiar with the use of drawing instruments. It is, in fact, necessary to do so; for the use of instruments is entirely a matter of practice, and can neither be taught by writing nor acquired by reading. For instance, it is most easy to write or to read the words, Rule a straight line; but if the line is long, it is by no means easy to rule it. All drawing instruments require not only some but a great deal of practice before they can be used properly, and the ruling pen, perhaps, more than any other. But beyond saying that its points must be kept scrupulously clean and sharp (they may be sharpened, when necessary,

on a hone); that in ruling with it, the two points must press evenly on the paper ; that the wrist must be kept perfectly stiff, and the motion given entirely from the shoulder and elbow, so that the pen may move parallel to itself, otherwise it will make a 'gouty' line, thicker in some parts than in others, and will probably tear up the paper as well ; beyond saying this, there is nothing in its use which can be learned except by practice and experience.

A description of the various instruments used in drawing, and an explanation of the manner of using them, would lead me far astray from my more immediate subject ; I would only, therefore, refer the young draughtsman to the excellent little work on mathematical instruments by Mr. Heather ;¹ and at the same time try to impress on him that not all that Mr. Heather, or anyone else, can write, will teach him to use the instruments ; he must have practice, he must have perseverance to go through with it.

I have also necessarily supposed the reader familiar with all the ordinary problems of geometrical construction, which are very fully treated of in many little works specially devoted to that study. The only construction which it seems advisable to give here is the exact method of setting off an angle, the ordinary methods either by the protractor or by the scale of chords not being sufficiently accurate for any delicate work.

At the point *A* in the straight line *AB*, to set off an angle α .

¹ Lockwood and Co., 1871.

To arrive at the construction, let us, in the first place, suppose that we have the angle drawn, namely, BAC .

With centre A , and any radius AB , draw the arc BC ; and draw the line AD bisecting the angle BAC .

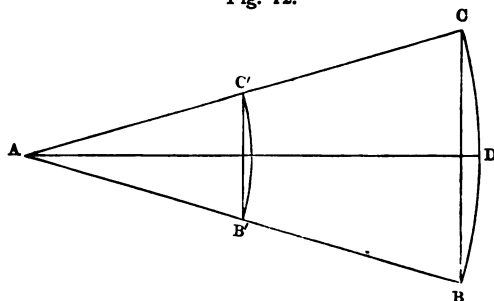
Then, since $BAC = a$, $DAC = \frac{a}{2}$.

Draw the chord BC , cutting the radius AD in d .

Then d is the middle point of BC , and $dC = \frac{1}{2}BC$.

With centre A and distance $AB' = \frac{1}{2}AB$, describe the arc $B'C$, and draw the chord $B'C$.

Fig. 12.



Then, by the similarity of the triangles $AB'C$, ABC , we have

$$\begin{aligned} B'C &= \frac{1}{2}BC \\ \therefore B'C &= Cd. \end{aligned}$$

Now if we call the radius, AB or AC , our unit, Cd is the sine of $\frac{a}{2}$; whence therefore we deduce the following rule :—

From a table of natural sines take out $\sin \frac{a}{2}$, which is

Cd. Then with centre *A* and distance equal to half the radius used for the table, sweep out the arc *B'C'*; and lay off on this the chord *B'C' = Cd*, just taken out of the table. Join *AC'*. *B'AC'* is *a*, the angle required.

Tables of natural sines are now seldom printed, and, when they are, are not sufficiently comprehensive; but a natural sine can be taken out of a table of natural versines with the most minute accuracy, though it is of course only on very large scales that seconds can be laid down.

$$\begin{aligned}\text{Since} \quad \text{vers } (90 + \theta) &= 1 - \cos (90 + \theta) \\ &= 1 + \sin \theta \\ \therefore \sin \theta &= \text{vers } (90 + \theta) - 1,\end{aligned}$$

or, the natural sine of an angle is taken out of the table of versines, by taking out the versine of the angle increased by 90° , and dropping the initial 1.

Natural versines are given in most tables for a radius 1,000,000; these may be reduced to any desired radius by ordinary proportion, thus,

$$1,000,000 : \left| \begin{array}{c} \text{given} \\ \text{radius} \end{array} \right| :: \left| \begin{array}{c} \text{natural} \\ \text{sine taken} \\ \text{out of the} \\ \text{tables} \end{array} \right| : \left| \begin{array}{c} \text{natural} \\ \text{sine as} \\ \text{required} \end{array} \right|$$

or natural sine required

$$= \frac{\text{nat. sine from the tables} \times \text{given radius.}}{1,000,000}$$

It will readily be seen that this reduction is much simplified if the radius chosen is an even multiple of 10; in

addition to this the radius should be of such length that its half, that is, the *effective radius* with which we describe the arc $B'C$, extends beyond the point to be fixed, or is longer than the line required to be drawn from A . Bearing these two points in mind, we may add that if the angle is small the effective radius should be as large as possible, but if the angle is large the radius may be more conveniently taken smaller.

Thus, for angles less than 30° , we may take our radius as 60 inches, and describe our arc $B'C$ with a radius of 30 inches. For angles between 30° and 60° , a nominal radius of 40, or an effective radius of 20 inches, will be as long as can be advantageously used; and for angles between 60° and 90° , an effective radius of 10 inches will be more suitable. But for angles greater than 60° , it is perhaps better to describe first an angle of 60° , by setting off the chord $B'C$ equal to the effective radius AB' ; and then, on the line AC drawing, in the manner just described, an angle equal to the excess. Thus, a longer radius may be used, and the construction is more accurate.

A fair amount of skill in the use of the instruments, and of proficiency in geometric drawing, may be attained by everyone, with practice and perseverance; but to complete a chart in a neat and workmanlike manner requires a certain amount of artistic talent. This, though capable of cultivation, is in the main a special gift; and just as many men, from some defect in their temper or eyesight, can never become first-rate surveyors, so there are many who, wanting this gift, can never become first-

rate draughtsmen. But supposing that the talent is not utterly wanting, it may, with assiduity, be cultivated to a sufficient extent ; for, after all, no very great amount of skill is required to pourtray the various features of the coast in a manner that may be at once neat and intelligible.

The instructions issued by the Admiralty for the guidance of surveyors explain so clearly the amount of finish which a chart ought to have, that I cannot close this chapter better than by laying them before the reader in a slightly condensed form.

The general object of a hydrographic survey is to produce such a chart that every portion of a sea-coast may be at once recognised by a stranger, so that he may without hesitation perceive the best manner of approaching or receding—of passing or quitting it—of maintaining a fit position in the offing—or of entering its ports without the aid of a pilot ; and all, under every state of weather, wind, and tide.

To produce a good chart, much more must be given than a mere outline of the shore with its general soundings and its off-lying dangers ; a complete chart should at once convey to the eye not only the relative position of its principal points and inflexions, but its varying *character*—whether springing abruptly from the sea in precipitous cliffs, or rising in bold acclivities, or in gentle slopes—whether broken into scattered points of rock, or throwing itself out in salient prongs and shelving ledges—whether its bays are encumbered with foul ground, or lined with

a smooth open beach—and especially whether it is easily accessible, with regularly decreasing soundings, or fronted by intricate banks and unconnected reefs, which, to pass through or to avoid, require a well-concerted series of sea-marks.

In such a chart the extent of the shoals, with the limits of the intervening channels, should be so manifest, the set of the tides so consistently expressed, and the directions so terse and simple, that by the rapid opening or closing of the marks, and the fullness and sequence of the soundings, the mariner should be able to trace the movement of his vessel in every fresh cast of the lead, and thus to feel his way with equal decision, promptitude, and confidence.

The topographic portion also of such a chart should be complete ; it should contain not only all the principal features which are conspicuous from the offing, but even the secondary objects which by their relative situations may serve to lead forward the eye to those that have been selected for sea-marks. And as it is of importance that these marks should be instantly discernible, some slight description might be annexed either to them or to the objects which serve to point them out ; for instance, suppose a clump of trees to be the cross-mark denoting the entrance of some winding channel, the chart should tell whether it consists of slender poplars, rigid firs, or of wide-spreading oaks ; again, if a church, or temple, or mosque, has it a dome, or square tower, or pointed spire, or minaret ; or, again, a series of cliffs may duly appear in the chart, yet how satisfactory to have some idea of

their relative heights, or to learn that certain parts of them are red, or white, or mossgrown, or that the strata are vertical, horizontal, or inclined.

In short, during the whole progress of a survey, it should be steadily borne in mind that, with whatever skill and labour it may be executed, it is only the means to an end ; and that its real merit will depend less on the science and taste which have been employed in its construction than on the practical utility of its results.

I would especially call the attention of the young surveyor and draughtsman to this last paragraph. Practical accuracy is the one important point ; though it may satisfy the scrupulous mind to work to exact seconds and tenths of feet, it may be borne in mind that it is difficult to lay down an angular difference of less than half a minute, and that even on the very large scale of 6 inches to a mile, 10 feet would be represented by the one-hundredth part of an inch, a quantity too small to show with any real pretension to accuracy. What the drawing instruments cannot draw ; what the straight-edge and the compasses will not enable the navigator to appreciate, is practically useless ; and the striving after a vain and impossible though pretentious perfection is mere waste of time. Let me not be misunderstood, however. I would by no means imply that because perfect accuracy is neither attainable nor necessary, a careless hasty manner of observing, or computing, or drawing, may therefore be allowed. On the contrary, any steps which can really increase the accuracy of the observations, the efficiency

of the instruments, or the rigorous correctness of the computations, must be adopted ; and in this the skill and practical knowledge of the surveyor is continually exercised.

The whole conduct of a nautical survey is, indeed, a constant struggle not only against the difficulties incident to any other survey, but against many peculiar to itself ; amongst which, besides those already dwelt on, is the altogether technical difficulty that the positions which require to be laid down with the greatest accuracy are themselves out of sight. It is the unseen shoal or rock which has to be most carefully guarded against, and it is the exact determination of such hidden dangers which is the most important and peculiar feature of nautical surveying, and which constitutes the real practical test of the value of any nautical survey.

CHAPTER IV.

OBSERVATIONS AFLOAT.

Method of Taking Soundings—Determination of Positions—Two Objects in One and an Angle—Two Angles—Danger Angle—Tidal Observations.

THE general soundings of any bay or confined space of water should, as a rule, be taken in straight lines nearly parallel to each other; but if the coast line should itself be nearly straight it is frequently preferable to take them in zigzags, making small angles with each other and nearly at right angles to the coast line, standing in, on each alternate course, as near as convenient, and out again as far as may be wanted, fixing the turning point of each course by exact angular observations.

Whichever method has to be used, whether the lines have to run in directions approximately parallel to each other, or in zigzag, in order that the boat from which the soundings are taken may steer a correct course, it is absolutely necessary that she should keep two fixed objects in one. So far as possible, these two fixed objects should be chosen as lying in the same direction, that is, both ahead, or both astern, so that they may both be seen at the same instant by the coxswain of the boat; and if one

of the two be near, the other very far off, the line can be kept with very great accuracy. The peak of a distant hill frequently affords a good further object ; and for the nearer objects, station poles can be set up along the shore, or other convenient marks made at suitable distances ; for these and for all secondary stations, white-wash splashes on a rock or large boulder are perfectly admissible, and their use will often save time and trouble ; but the whitewash must be put on so that the marks may be readily distinguishable one from the other.¹

When, as will sometimes happen, it is necessary to fix the boat's course by objects, one ahead and one astern, it is not quite so easy to keep on the correct line, but the line must be kept as closely as possible. To facilitate this an instrument was devised by the late Lieut. Raper, which by a pair of reflectors shows the object astern as though it was ahead ; it is doubtful, however, whether its advantage is not more apparent than real—so much distinctness of sight is lost by the double reflection, that it is necessary to be more careful in choosing the stations. The instrument itself is always liable to become useless in consequence of the reflectors getting wetted ; and, more than all, if the observer's attention is occupied with this instrument, he is not ready to fix positions by exact observations with the sextant : one instrument is perhaps quite as much as he can attend to, and in order that the

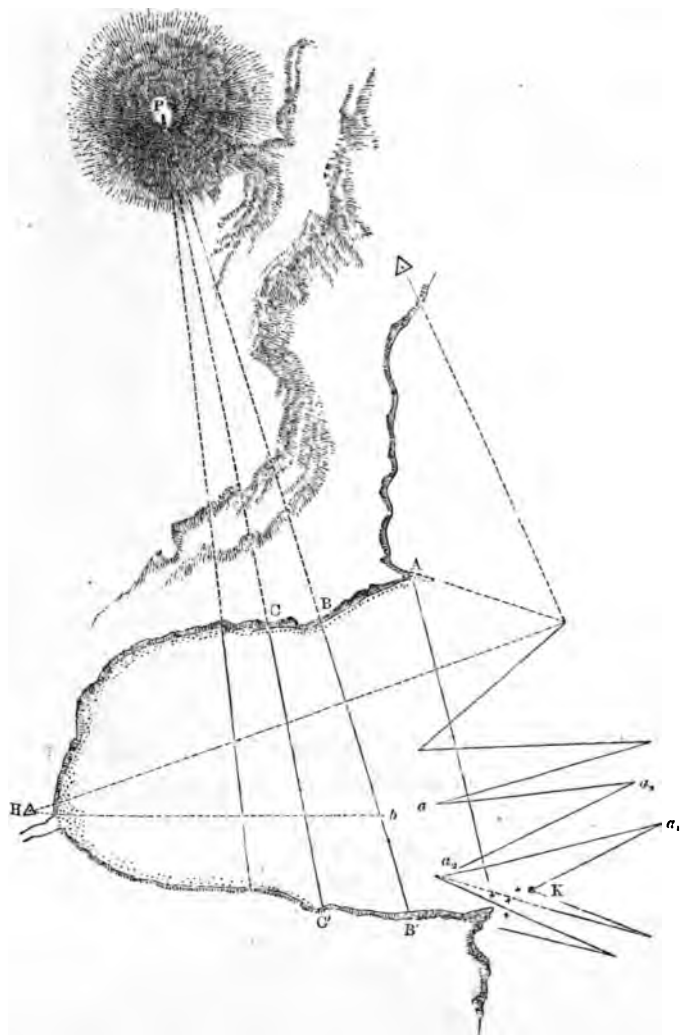
¹ 'In whiting a spot much time is frequently saved by first wetting the spot well, and then dredging it [the dry lime] on by hand. It stands better, and less is lost by *streaming* off—the thickness can be laid at pleasure, and is *instantly white*.'—BELCHER.

soundings may be laid down accurately, the sextant must be in almost constant use.

When the two objects chosen as the leading marks of the course are correctly in one, the position of the boat at any time is exactly fixed by one angle between the known line of the boat's course and any station which has been laid down on the chart, taking care only that the angle be such as to give a good 'cut,' that is to say, ranging between 60° and 120° ; by laying off the observed angle on the given line, and moving the parallel ruler on till it passes through the observed point, the position of the boat is at once fixed. Suppose, for instance, *AHK* to be a bay in which soundings have to be taken. Let *P* be a distant peak; and the more distant the better, so long as it is distinctly visible. The secondary stations *B*, *C*, &c., being marked in any way most convenient, the boat must take care, in whichever direction it is going, to keep its individual station on with *P*, so that at every point it may be on one of the known lines *B'B*, *C'C*, &c. Suppose the boat to be sounding on the line *B'B*, and at a position in that line the angle between *B* and *H* to be taken, $73^\circ 30'$. Set off an angle $73^\circ 30'$ on *B'B*, towards *B*, and with the parallel rulers run the line, just set off, on till it pass through *H*, as *Hb*, then *b* is the position of the boat at that particular sounding; and in a manner precisely similar, any position on any of the leading lines may be very exactly determined and laid down.

The accuracy of this determination of the position of the boat depends of course on the accuracy of the observed angle. As, however, a small error may occur in

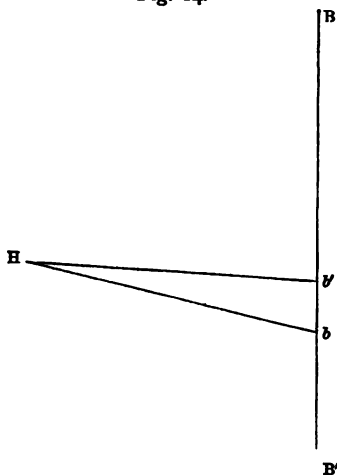
Fig. 13.



the observation, it is necessary to note the conditions under which it produces the least effect on the result.

Let the angle observed at the boat between B and H be α' instead of the true angle α , being increased or diminished ϵ by the error of observation : and let the position as determined by the observed angle α' be b' , instead of the true position b .

Fig. 14.



Then the error in the position arising from the error in the observation is bb' and ϵ the error in the angle is $\alpha' - \alpha$, or bHb' .

Now

$$\frac{bb'}{bH} = \frac{\sin bHb'}{\sin b'Hb'}$$

$$bb' = bH \cdot \frac{\sin \epsilon}{\sin \alpha'}$$

or,
$$bb' \propto \frac{bH}{\sin \alpha},$$

if ϵ is considered a constant quantity :

that is, for a certain small error in the angle observed, the corresponding error in the position becomes *greater* according as H , the observed point, is further off, and also according as the observed angle becomes smaller.

Since
$$bb' = \sin \epsilon. \frac{bH}{\sin \alpha},$$

if $\alpha = 90^\circ$,
$$bb' = \sin \epsilon. bH,$$

the least value it can have whilst bH remains constant ;

$$\begin{aligned} \therefore bb' &= \sin \epsilon. \frac{\frac{1}{2} bH}{\frac{1}{2}} \\ &= \sin \epsilon. \frac{\frac{1}{2} bH}{\sin 30^\circ}; \end{aligned}$$

that is to say, the error is as little when the observed angle is only 30° , as when it was 90° , if the distance of the observed point is half the previous distance.

Hence we have the simple rule ; the position observed should be as near as possible, and the angle should be not very far from a right angle.

It is not, however, necessary to fix each sounding in this exact manner : if the soundings are taken at regular intervals, and every third or fourth determined by the observation of an angle, the intermediate ones can be put in at equal distances between. But as soon as taken, they must be entered in the note book, in the fullest detail ; the depth, nature of bottom, time, and angular

observations, must all be carefully written in at once ; nothing is to be trusted to memory ; and amidst such a very great number of observations, in order to avoid confusion in afterwards laying them down, it is absolutely necessary to establish a regular system of entering them ; as long as the system is regular and easy to be understood, its particular form is of very little consequence.

Small matters of detail are in practice often of great importance ; and amongst these may be counted the necessity of entering the observations in a neat and legible manner ; a metallic pencil should always be used, as its writing does not rub or wash out, as that of a common black-lead pencil does, and the sight book may at times be exposed to rough usage and to wet.

If no irregularity in the soundings should occur, the whole of the work, so far as relates to the bay, may be done in straight lines, in the manner just described ; but in all cases, the boat should be provided with a buoy and grapnel ready to be thrown overboard on any suspicious cast being made. Soundings should then be taken with care and frequency in this immediate neighbourhood, starting from the buoy in every direction, till the full detail of the shoal (if it exist) has been ascertained, or till its non-existence is satisfactorily proved ; and this can be done either at once, or afterwards, as may be most convenient ; the position for resuming the line of soundings, or returning to the suspected danger, being, in either case, marked by the buoy.

This closer and particular examination necessarily carries the boat out of the chosen lines ; and the positions

of the several soundings have to be determined by independent observations. Again, a rock may be found outside the bay, where it is inconvenient to refer it to a guiding line ; and if, in order to take soundings well out to sea, the boat has to make a series of stretches seaward, as denoted by the zigzag Ka_1, a_1a_2 , &c., it is necessary to fix the positions at which the boat turns. All isolated positions of this nature, whether of dangers, or mere turning points, have to be determined by two angles, one of the observed stations being the same for each of them : such two angles are enough to fix the position ; but in all cases when the position is important, a third and even a fourth angle should be taken as a check on the first observation. The angles having been noted, the position can be laid down by means of the station pointer, which is accurate enough, if merely the boat's turning-point has to be marked ; but is scarcely so if the position is a rock or danger of any kind : a more exact method must then be adopted, a method which is very frequently called 'The Problem of the Two Circles.'

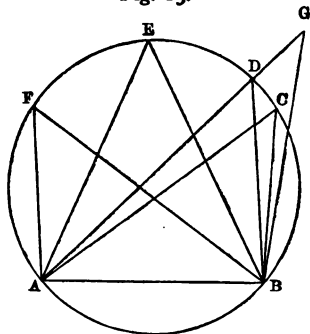
The geometrical fact on which it depends is this ; that if from any two points, as A and B , in the circumference of a circle, pairs of lines are drawn to any other points in the circumference, on the same side of AB , as C, D, E, F ; these lines all contain the same angle ; that is, the angles ACB, ADB, AEB, AFB , and as many more as can be drawn in the same way, are equal to each other. (Euclid III. 21).

This being the case, it is evident that if the points A and B are fixed, the circumference AEB is the measure

of a constant angle, which is spoken of as 'the angle in the segment AEB ;' and any point at which AB subtends this angle must lie on the circumference AEB .

For if not, let it lie beyond it, as at G ; join AG , cutting the circumference in D ; and join BG , BD .

Fig. 15.



Then, by the supposition, the angle AGB is equal to the angle in the segment AEB ;

but, by construction, ADB is an angle in the same segment;

therefore, the angle ADB is equal to the angle AGB ;

or, the exterior angle of the triangle BGD is equal to one of the interior and opposite angles;

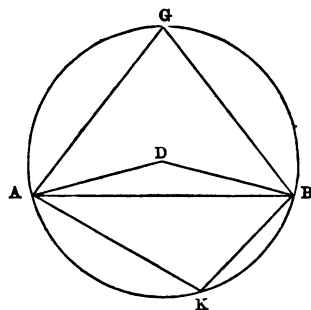
which is impossible. (Euclid I. 16).

Therefore the point G , at which AB subtends this constant angle, cannot fall without the circumference; and it may be proved, in exactly the same way, that it cannot fall within it. It, therefore, must lie on the circumference.

If, then, the angle between two known points is observed, the point from which the observation is made is determined to this extent ; it lies on the circumference of the one only segment, containing the observed angle, which can be drawn on the given base. It is shown, in Euclid III. 33, how to draw this segment. Another way, simpler in some respects, is as follows :

Let A and B be the two known points ; and let the angle between them be observed, 72° ; it is required to draw the segment of the circumference on which the point of observation lies.

Fig. 16.



At each of the extremities A and B of the straight line AB , set off the complement of the observed angle, that is, in this case, $90^\circ - 72^\circ$ or 18° .

Let the lines making these angles meet in D ;¹ then

¹ When, as is frequently the case, the complements set off are very small angles, these lines meeting in D make a very bad cut, and the point D has to be accurately fixed by drawing a perpendicular from the middle point of AB , as shown in figure 17.

AD is equal to BD , and a circle described with centre D and distance DA , will pass through B .

The segment AGB is the segment required.

It is proved (Euclid III. 20) that the angle at the centre of a circle is double the angle at the circumference, standing on the same base, or part of the circumference ; that is, the angle ADB is double of the angle AGB .

Now since the three angles of a triangle are together equal to 180° ; and since in the triangle ADB the angles at A and B are each equal to 18° , or are together equal to 36° ; it follows that the angle ADB is $180^\circ - 36^\circ$ or 144° ,

and the angle AGB is half of the angle ADB ,
therefore the angle AGB is 72° , which is the observed angle.

The point of observation is therefore *somewhere* on the circumference AGB .

If the observed angle be greater then 90° , drop 90° and lay off the remainder : if, for instance, the observed angle is 108° , lay off $108^\circ - 90^\circ$ or 18° . The construction is then made exactly as before ; but the position will now lie on the circumference of the small segment AKB .

The proof of this is very simple.

$AGBK$ is a four-sided figure inscribed in a circle, and any two opposite angles are, therefore, together equal to two right angles : (Euclid III. 22)

therefore the angles at G and K are together equal to 180° :

and we have shown, in the former case, that the angle at G is 72° ;

therefore the angle at K is $180^\circ - 72^\circ$, or 108° .

In making the construction, care must of course be taken to draw the segment wanted on the proper side of AB , according to the bearing of A and B , which is roughly known ; for it is evident that the figure could be entirely reversed, so that the segment AGB should be below AB . To ensure this being done, without chance of mistake, it must be remembered that,—

If the observed angle is *less* than 90° , the required position is in the large segment ; and if the observed angle is *greater* than 90° , the required position is in the small segment.

If the angle should happen to be actually 90° , the segment required is a semi-circle, which may be drawn at once on the given line as a diameter, taking its middle point as the centre.

When the required position has been determined to the extent just shown, its determination is completed by repeating the construction on the base joining the other pair of observed points, one of which, as has been already said, must be common to *both* observations. If, for instance, the angle between B and C was observed to be 40° ; (fig. 17) the angles CBE , BCE are set off, each equal to $90^\circ - 40^\circ$, or 50° ; and the circle described with centre E , and distance EB or EC , gives BLC , the segment required.

The position of observation is then on the circumference BLC ;

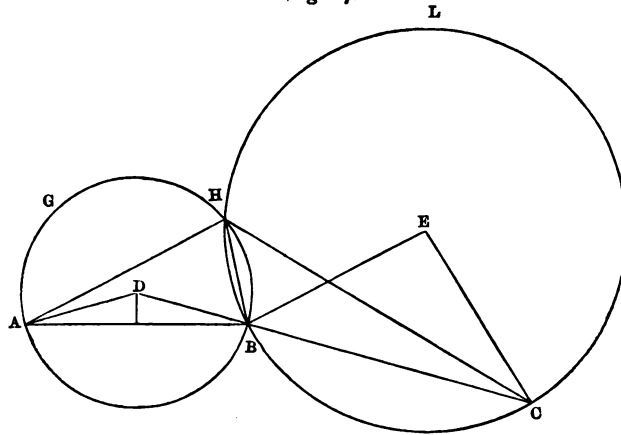
and we have before shown that it is on the circumference AGB ;

it must therefore be H , the intersection of the two

circles, the only point which satisfies the double condition.

The position is thus completely determined, and, in cases of great importance, by observing angles from *A* or

Fig. 17.



B to other points, and repeating the construction ; but this is seldom needed. In order however that the result of these constructions may be accurate, great care is necessary not only in actually making the observations, but in selecting the objects to be observed. Taken in a small boat, it cannot be expected that the angles are quite exact ; and the points observed must be so chosen as to reduce the effect of any small error in the observation to a minimum.

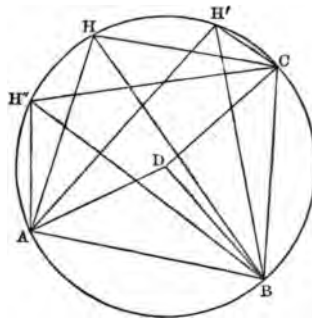
To do this, the circles must make a good cut, that is, they must intersect nearly at right angles. A neglect of

this consideration leads to the 'Impossible Case,' and to others so nearly resembling it as to render the determination of the point quite worthless.

The 'Impossible Case' is when the position of observation lies on that circle which passes through the three observed points. If three points are not in a straight line, a circle can always be drawn, passing through each of them : the centre of this circle is very simply found, thus :

Join the three points, so as to form a triangle ; bisect any two of the sides of this triangle, and draw perpen-

Fig. 18.



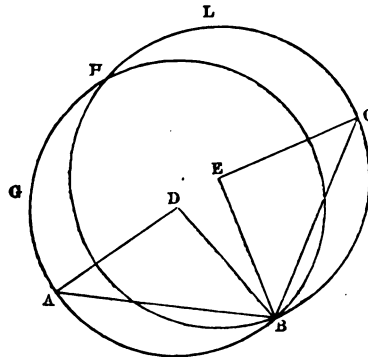
diculars to them from the points of bisection : the point in which these perpendiculars meet, is the centre of the circle which passes through the three points. (Euclid IV. 5.)

Thus if A , B , and C be the three points chosen, and from H the angles AHB , BHC , are observed, it is quite clear that the position H is not determined ; since from

each part of the construction we merely arrive at the fact that it lies *somewhere* on the circumference AHC ; so that it may be anywhere on this circumference, as at H or H' or H'' .

It is worth noticing that if the position is laid down by means of the station pointer, or, as is often done, by protracting the angles on a piece of tracing paper, and moving it about on the chart till the three lines so drawn pass through the three points, this incompleteness of the determination, when it occurs, is not necessarily shown; for a point that satisfies the observation having been pricked off, there is no construction to call attention to the fact that a very great many other points would also

Fig. 19.

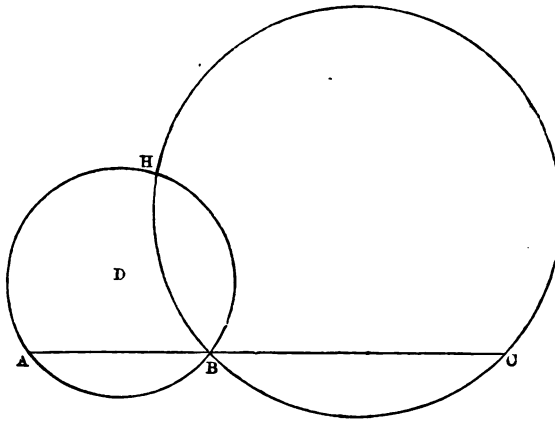


satisfy it, and that the position is quite indeterminate. The mistake, if once made, may thus remain, damaging the correctness of the work more or less seriously, according to the importance of the position.

But without making exactly the impossible case, which could happen only rarely and by a curious agreement of the angles, the segments described on the lines AB , BC , may easily lie so near each other as to leave the point practically as undetermined as in the impossible case. Thus in figure 19, the intersection of the circles AGB , CLB , can scarcely be considered as determining the position H .

In practice, it is quite easy for the eye to carry round the circle through the three points thought of; and if

Fig. 20.



this seem to pass anywhere *near* the observer, one of the proposed points must be at once rejected. A familiar knowledge of the relative positions of objects which must be avoided, or which should be preferred, will, however, save trouble, and do away with the chance of mistake.

Fig. 21.

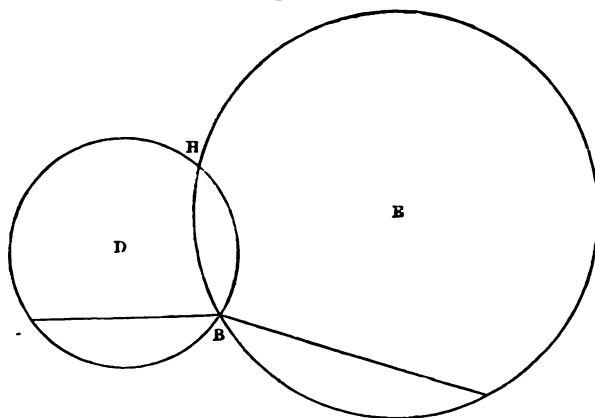
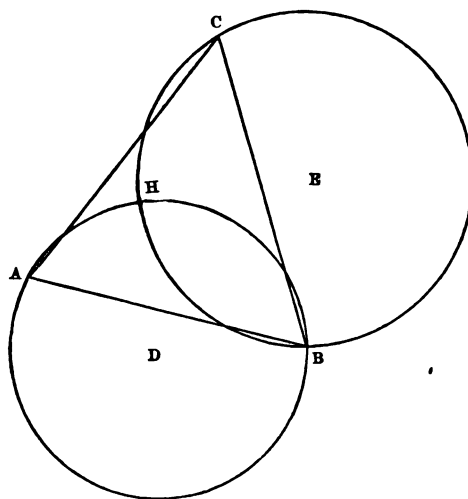


Fig. 22.

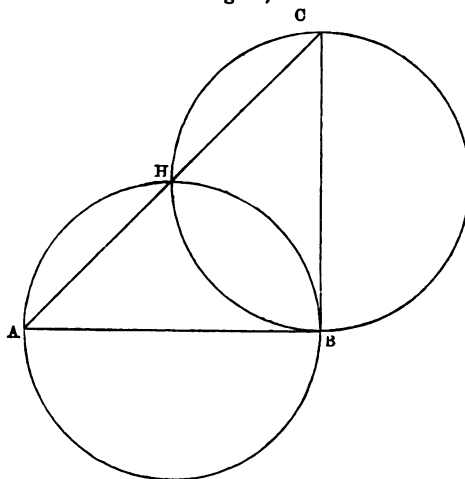


Cases resembling the impossible case only occur when the centres of the two circles fall very close together, which cannot possibly happen unless the angle ABC , towards H , is less than 180° . If AB , BC , are in the same straight line ; or if B , the middle point of the three, is on the *nearer* side of the line joining AC , the impossible case, or anything at all like it, cannot result. One of these two conditions may therefore be sought for ; there is only one other which ensures safety ; that is, when the point of observation lies clearly within the triangle formed by joining the points ABC . These three cases are shown in figures 20, 21, and 22 ; and it will be at once seen that in none of them can the difficulty spoken of arise.

But more generally, depending on the rule, observed in the protraction, that the centres of the circles must not be near each other, it is sufficient to notice that the two circles are very different either in situation or size. Thus if the lines joining AB , BC , are nearly at right angles to each other, as in figure 23, and the three points are nearly at equal distances from the observer, the intersection of the two circles will be very good, and it will be good in almost all cases where the centre of each circle is without the other circle. This can be ensured almost without further reference to the position of the three points, if *two* of them are near to, and, roughly speaking, equidistant from the position of observation, at which they subtend an angle not very different from a right angle ; but the *third* is as far off as possible, and the angle between it and the middle point is, in comparison, small. But it is ob-

jectionable to have the middle point near, and the other points both far off ; for in this case, the two circles will very nearly touch externally, so that their intersection is very badly determined.

Fig. 23.



If it is noticed that as the boat changes its position, the angle between two objects changes rapidly, whilst the angle from one of these to a third changes slowly, these points are suitable, and the circles will have a very good cut : for if the one angle is changing rapidly, the boat's course must be nearly at right angles to the circumference of the one circle ; and if the other angle is changing slowly, the boat's course must, at the time, nearly coincide with the circumference of the second circle ; thus the same line being, roughly, a normal to one

circle and a tangent to the other, the two circumferences are very nearly at right angles to each other.

Bearing these general principles in mind, a very little practice will enable the observer at once to see what points will give a satisfactory projection ; but it is absolutely necessary to be guided in this way ; for whether the position is laid down on the chart by the actual protraction of the angles and the circles, or, more readily, by the station pointer, any ambiguity which would be shown by the one method really exists also in the other, and is more insidious, because it is not brought prominently into notice.

When soundings of a more extended nature are being carried on from the ship—as, for instance, in taking the deeper soundings at some distance from the coast—it is often sufficiently accurate to fix them by careful cross-bearings, the standard compass on board being much more trustworthy than a similar instrument moved to different localities on shore. These bearings should, of course, cut nearly at right angles ; but when, as in this case, minute accuracy is required, it is better to take four bearings making angles of about 45° with each other ; the intersections of these give two nearly perpendicular cuts, as well as the others at 45° , which serve as valuable checks on the observation.

But, after all, surveyors habitually mistrust the compass ; and, for any surveying work, positions are more certainly laid down by the method just described, of observing the angles subtended by two pairs of adjacent points.

If a position is so determined, a line of soundings may be run from it by compass and patent-log ; every opportunity being taken to check, not only the compass, but the steering, by the accurate determination, in the same way, of intermediate positions, and especially of the final position, at which the line ends.

If night should close in before the work is finished, so that three satisfactory points on shore cannot be made out, it is often possible to utilize a star as the distant point, if two light-houses, or known lights, can be used as the two nearer objects.

To determine the ship's position in this way, the altitudes of the star, and of the lights (if they have any), must be observed, as well as the angles between the three objects. The horizontal angles—that is, the differences of bearing between the lights and the star—are thus readily found. The star's true azimuth is also calculated, and is drawn as a line of bearing through the observer's estimated position, which can be laid off with sufficient accuracy by the cross-bearings of the two lights. From each of the two points on shore, lines are then drawn making the observed angles with this line of bearing, and the intersection of these two lines is the ship's position.

Thus, let A, B , be the two lights observed, making with the star S the *horizontal* angles BKS, AGS ;

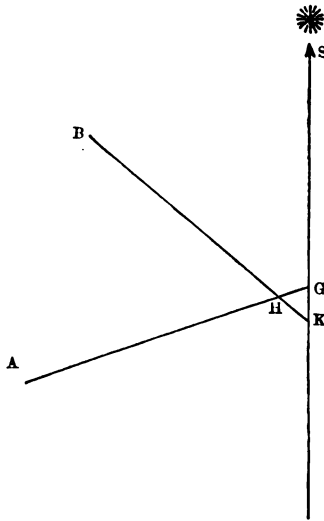
Draw KS , the line of the star's true azimuth, through the estimated position ;

And through A and B , draw the lines AG, BK , making the observed angles with the line KS .

Then H , the intersection of these two lines, is the position required.

Clearly, the angle AHB must be such as to give a good cut ; and it is better if the angles observed also give good cuts at G and K ; it would therefore be well that

Fig. 24.



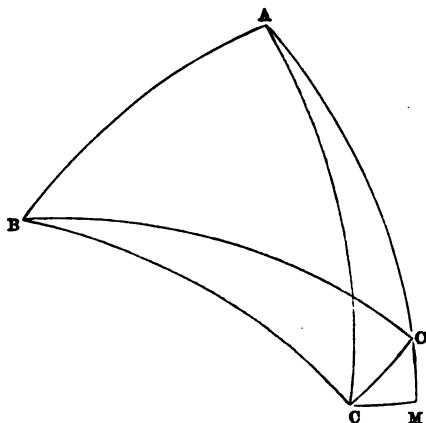
none of the angles BKS , AHB , and AGK differ very much from 60° .

A moment's consideration will show that this method of determining a position is, in fact, a determination by cross-bearings, accurately observed and calculated ; and the position may be laid down, with a slightly different construction, by applying the corrected horizontal angles

to the star's azimuth, so as to find the true bearing of each of the lights ; these lines of bearing drawn through *A* and *B* will, of course, be identical with the lines *AG*, *BK*, drawn as already described, and their intersection determines the position of observation.

As affecting the determination of a position in this manner, it is necessary to call attention to the accuracy attainable in the star's azimuth ; which being calculated from the estimated latitude, and from the altitude observed to a dark, perhaps a hazy sea horizon, is subject to the error arising from probable small errors in these elements.

Fig. 25.



As in each of these cases, the erroneous side (whether the co-latitude, or the zenith distance) is adjacent to the angle affected, the trigonometrical principle involved is the same, and the investigation can be enunciated in one, namely :

In any spherical triangle, to find the error in an angle, which arises from an error in *one* of the adjacent sides.

In the triangle ABC , let the side AC be thrown out of its place, by an error in its length, into the position AC' ; AC' being the erroneous value of the side AC .

Then the angle CAC' is the error in the angle A , arising from the error in the side,

And if a small circle be drawn round A as a pole, passing through C , and cutting AC' in M , MC' will be the error in the side AC .

Again, by the supposition that the other sides are unaltered, BC is equal to BC' , and a small circle described round B as a pole, and passing through C , will also pass through C' ;

The angles BCC' , $BC'C$ are therefore each right angles,

As also are the angles AMC , ACM ;

And therefore the angle MCC' is equal to the angle ACB , each being the complement of ACC' .

Now, since CM is the arc of a small circle,

$$CM = CAM. \sin AC$$

$$\therefore CAM = CM. \operatorname{cosec} AC.$$

But since CMC' is a very small triangle, no appreciable error will arise from considering it as a right-angled plane triangle, and

$$\therefore CM = C'M. \cot MCC'$$

$$= C'M. \cot ACB$$

$$\therefore CAM = C'M. \cot ACB. \operatorname{cosec} AC; \text{ that is,} \\ \text{error in } \angle A = \text{error in side } AC. \operatorname{cosec} AC. \cot C.$$

Applying this to the particular cases which we have under consideration,

error in azimuth

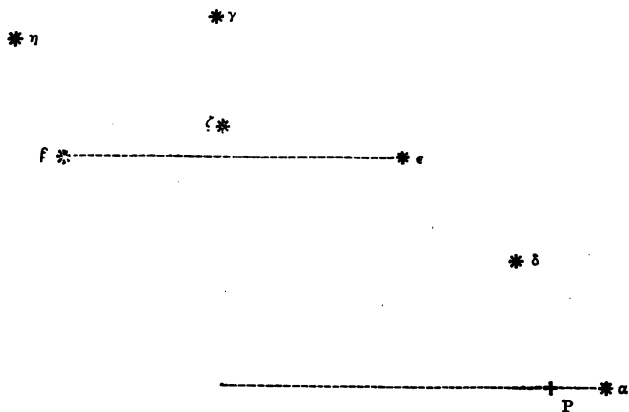
$$= \text{error in latitude} \cdot \sec \text{latitude} \cdot \cot \text{hour angle}$$

and also

$$= \text{error in altitude} \cdot \sec \text{altitude} \cdot \cot \text{angle of position.}$$

Hence, since $\cot 90^\circ = 0$, if the hour angle is 90° , on either side of the meridian, the error in azimuth arising from error in latitude vanishes; and similarly, if the angle of position is 90° , the error arising from error in altitude vanishes. So far as possible, therefore, the star's hour

Fig. 26.



angle and angle of position should *both* be nearly 90° ; a condition which may generally be attained with some one of the stars near the pole. In northern latitudes, if the pole star has an hour angle a little less than 90° , on either

side of the meridian, nothing better can be found ; but when its hour angle is not suitable, some other of the nearest circumpolar stars should be made use of. Without reference to the 'Nautical Almanack' for the pole star's hour angle, it can always be told approximately by the position of the Little Bear. If the two stars β and ϵ Ursæ Minoris are in a horizontal line, the pole star's hour angle is very nearly 90° ; and it is worth noticing that at this time (when β and ϵ are horizontal) the altitude of the pole star is also the altitude of the pole—that is, is the latitude ; which can thus be found at once, by observation alone, without any computation beyond correcting the altitude, and with an accuracy quite sufficient for the ordinary purposes of navigation.

But it is not only in operations connected with surveying that it is necessary to determine a ship's position with rigid accuracy ; and important as compass observations are, those made with the sextant are always of great value, and are even more to be relied on if the determination of the positions on shore is certainly trustworthy.

As to this, the navigator is called on to examine into the value of his charts ; and the age of the chart, or its nationality, the name of the surveyor, its being a single work, or a compilation, must guide him as to the more or less thorough dependence to be placed on it. A compiled chart must be used with great circumspection, especially so far as relates to the distance between points and leading marks ; but the lines of bearing are more likely to be tolerably correct. Old charts, even by cele-

brated surveyors, were drawn without that attention to strict accuracy which is now enforced : thus, during the very close survey of the coast of Sicily which has been carried out during these last few years, the former survey by Admiral Smyth was found in many instances extremely incorrect, and that often (as it seemed) intentionally ; rocks and dangers being marked sometimes as much as half a mile to seaward of their real position, with the view, perhaps, of frightening ships into keeping a good and safe offing : but whether errors are due to intention or to inaccuracy, the effect, so far as we are here concerned, is the same, and the distances in old charts cannot be trusted. But with a chart of a recent, close and single survey, in which the distances are marked in with as scrupulous accuracy as the lines of bearing, the most exact determination of the ship's position can be made by means of observations with the sextant, and the practised observer will find no more difficulty in using the one instrument than in using the other. The advisability of employing such methods must be settled by the navigator ; but supposing that he is provided with a chart on which he *knows* he can in every way depend, the sextant offers many advantages over the azimuth compass. In addition to those already spoken of—the superior correctness of the observation, the closer accuracy of the reading off, and the absence of an uncertain and frequently changing deviation—there is another, which in the present day assumes a new importance, in consequence of the extreme length of many of our ships. A length of three or four hundred feet is itself a base line which introduces totally

new considerations into the determination of a ship's position ; and the bearing of a point on shore, as taken from the standard compass aft, may be very different from what it would be if taken from a station well forward. Now, the standard compass cannot be moved ; the sextant can ; and observations may be taken with it from either end of the ship, according to circumstances. In intricate navigation, the bow is clearly the part of the ship first to approach any danger, and from the bow the observations necessary to indicate its distance ought to be made.

Navigators are in the habit of trusting exclusively to the lines of direction as shown by leading marks ; but they frequently do not pay sufficient attention to the necessarily vague wording of the sailing directions, or of the notes on the chart. Tower *A* just open of point *B* clears shoal *C*, is a very common form of words, perfectly correct in the sense in which the surveyor writes it down, but liable to misconstruction. A rash navigator will consider *A* just open of *B* if *A* is at all visible : a very cautious navigator, on the other hand, will not consider it open till he can see a wide gap between the objects named ; and though the more cautious plan is undoubtedly the most sure, still, circumstances may very well arise when it is necessary to steer the shortest, closest course consistent with safety ; not to speak of the constitutional temperament of many men who would chafe at the very idea of making a needless round.

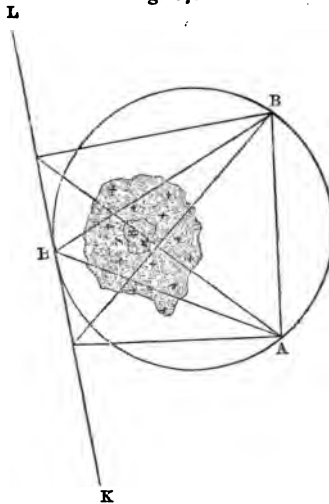
But it is frequently, nay, commonly impossible to word the sailing directions with closer accuracy ; for clear leading marks can but seldom be found absolutely coin-

ciding with the nearest line which may safely be sailed along ; so that the exact interpretation of the phrase used is necessarily left to the judgment and discretion of the navigator. This necessity may, however, be in great measure avoided by a more rigorous determination of distances and positions. Thus, on approaching a known but hidden danger, the ship's position may be laid down accurately on the chart ; and the doing so with a station pointer will give very little trouble to a well-practised hand. From this position the ship's proposed course can be drawn on the chart, in accordance with the sailing directions ; but having resolved at what distance the danger should be passed, safety may be almost ensured by constant attention to what is called the 'Danger Angle,' a method which is practised indeed by surveying ships, but is little known or used in the general line of the service. There is, however, nothing to prevent its universal adoption ; the observation itself, and its immediate application, are both perfectly simple and easy ; and though the determination of the angle or series of angles is a little more complicated, it can be done leisurely beforehand, or may indeed be marked permanently on the chart, in the same way as the lines through leading marks now are, and with an entire absence of all that ambiguity or uncertainty of expression which, slight as it really is, does at times prove a source of confusion and danger.

With the view to thus laying down on the chart, or entering in the sailing directions, the danger angle of any shoal to be cleared, I proceed to discuss the principles of its determination, which are indeed based on the pro-

perty of the circle which has occupied us for the last twenty pages; namely, the equality of angles in the same segment. If S denote a shoal, and A, B , two known positions on shore, then, if a segment of a circle, AHB , be drawn on AB , passing at the nearest safe distance beyond S , the angle in that segment is called the danger angle. So long as the angle between A and B observed from the ship is less than this angle, the ship is perfectly safe; if it is equal to it, there is need of great

Fig. 27.

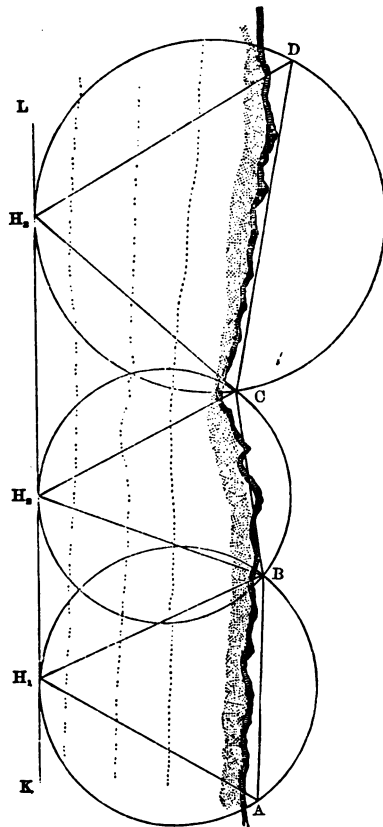


caution; if it is greater than it, there is DANGER. Under no circumstances, therefore, unless with the deliberate intention of running the ship aground, should the angle be allowed to get greater than that laid down as the danger angle.

If, as shown in fig. 27, the coast line is nearly straight, and two suitable objects can be found in the relative position of A and B towards S —that is, one well on each side of the shoal—the segment, as AHB , can be drawn without any difficulty, and the course laid in the proposed direction, as a tangent to the circle.

But if, as not unfrequently happens when the coast is low and sandy, the shoal runs for a considerable distance nearly parallel to the shore, but not reaching to any great distance from it ; if, for instance, it is the extremely flat nature of the bottom, which, even some miles off, does not give sufficient water for ships of any burden ; a succession of marks, A, B, C, D , &c., must be chosen, each giving a new danger angle, which will be put on the sextant successively, as the old marks pass astern. In this there is no difficulty : as each new object comes well in sight, the new angle should be observed with a second sextant, so as not to lose the former angle till the second of its marks is passed. Thus, in making the course KL , the angle AH_1B is kept to, till A is clearly on the quarter, or, till, in keeping the straight course, the angle between A and B begins to get smaller : this shows that the point H_1 is passed ; the angle between B and C should then be observed with a second sextant, still continuing to observe the angle between A and B . As the ship proceeds on her course, the angle between B and C gets gradually larger, and more and more nearly equal to the angle BH_2C , which is the danger angle for these two points. When the angle begins to get smaller, we know that H_2 is passed ; the angle between A and B may

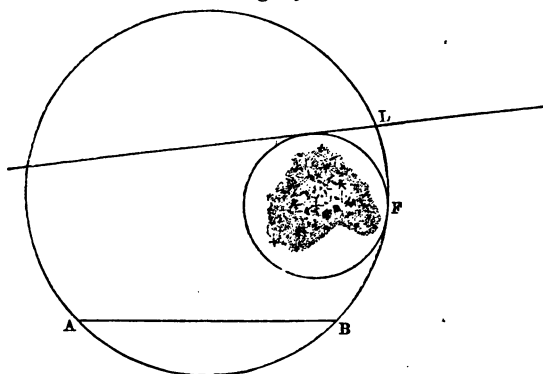
then be lost sight of, and the angle between *C* and *D* be observed ; and a continuation of the method will carry



the ship, in perfect safety, along any length of shoal near the shore, whether straight or curved.

It will be noticed, however, that the line joining the marks observed must not run *towards* the shoal. The best position for them is, as shown in fig. 27, when the perpendicular bisecting AB runs nearly through the middle of the shoal; or, as in fig. 28, the part of it cleared by the angle between A and B . Considerable latitude may be allowed in the choice of objects; but fig. 29 shows

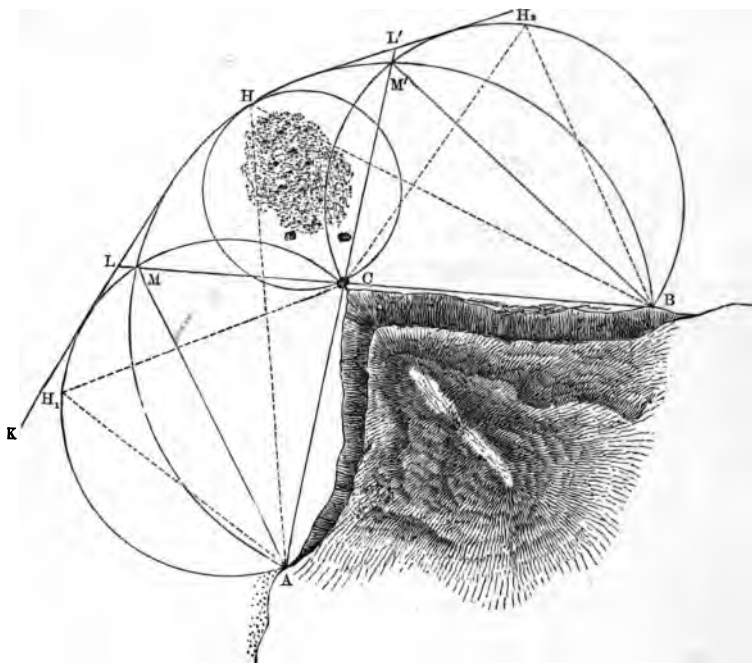
Fig. 29.



the inconvenience of points which differ widely from this rule: a circle such as ABF , the smallest which can be drawn through A and B , clearing the shoal, is evidently of very little practical value to a ship which wishes to follow the course KL , a line which is perfectly safe, being altogether outside the danger circle—that is, the smallest circle which can be drawn round the shoal, containing every danger of the shoal, on the seaward side, well within its circumference. Marks, then, in the relative position of A and B cannot be used, and others must be chosen.

Practically, there is little or no difficulty about finding satisfactory objects. The leading marks of a straight course must lie absolutely in one particular direction ; one of them should also be at a considerable distance ;

Fig. 30.



and such are very commonly not to be met with ; but the restriction on the choice of suitable objects, for this circular method, is so trifling, that a case can scarcely arise in which the marks do not readily, almost naturally, present themselves.

The only case, indeed, which admits of any difficulty is that shown in fig. 30. The shoal S is here at the extremity of an elevated promontory ; and a ship approaching it by courses such as KL or $K'L'$ cannot see marks in a suitable position for clearing it. Select two good marks, as A and B , one on each side of the headland, and each clearly to be seen from C , its extreme point. On the line AB draw AHB , the danger segment—that is, the segment which touches the danger circle on the seaward side. Draw the lines BC , AC , cutting the circumference AHB in the points M and M' ; and join AM , BM' .

The angle AMB is the angle in the segment AHB —that is, is the danger angle ; but it is also the angle between A and C . A ship, therefore, steering along the line KL may safely be guided by observing the angle between A and C , and considering the segment AHC (the segment described on AC , containing the danger angle) as a *secondary* danger segment, *until B can be seen*. But as soon as B can be seen—that is, when the ship is in the position L —the true danger segment can be followed, LHL' , until the ship arrives at L' , and A is lost behind the land ; the other secondary danger segment, CH_2B , then comes into use, till the danger is well passed.

These secondary segments may not always be absolutely necessary ; for, as in the figure, the mark round the corner may be seen before the danger is very close ; but in cases such as this they are always advantageous, as serving to keep the ship at a proper distance from the land—~~that~~ when the second mark opens, she is in a

position to make the shortest course round the shoal, as given by the curved line *LHL'*.

In closing this branch of the subject, I would again call attention to what I have before said, that surveyors, strictly speaking, have nothing to do with the compass, except to determine its errors. I am writing here from a surveyor's point of view. I have wished, in these last few pages, to show how the position of a ship, when in with the land, may be laid down; how dangers may be avoided; how a course may be steered, *without* the compass. In the practice of navigation and pilotage, the attempt to do without the compass would be worse than absurd; it would be blameable in the extreme; but cases may occur, as I have endeavoured to point out, in which the sextant is a safer guide than the compass, or in which it is a most valuable auxiliary to it. To examine into these cases is the navigator's duty; it is the surveyor's duty to provide him with the necessary data, accurately laid down.

All harbour and shoal soundings, before they can be laid down on the chart, must be corrected for the height of the Tide; they must be reduced to what they would have been, had they been taken at Low Water, Spring Tides. The necessity of this is obvious, for a navigator when clearing a shoal, entering a port, or taking up an anchorage, must be able to see at a glance what is the least depth which he can possibly find.

The reduction cannot be done correctly without an ex-

tended series of tidal observations, but observations continued through a fortnight, that is, through the whole tidal range, will be often sufficient, and when time is short, observations during a day or two at spring tides will give very useful results.

The observations consist simply in noting, at frequent intervals, the height of the tide on a vertical scale of feet painted either on a pole set up for the purpose, or on the face of a rock. Such a scale is called a 'Tide Gauge,' and is divided into feet and half feet by distinct black lines on a white ground. As no reading less than half feet need be attempted, the black lines should be painted so as to be clearly seen through a glass, at some distance; and if the gauge can be set up so that the day observations can be made from the ship, it is the more convenient; but this is not always possible. In anything like a close harbour, a deep bay or fiord, such as those which indent the Western Coasts of Ireland, Scotland, or Norway, or in the estuary of a river, the height of the tide is very different, in different parts, at the same time; and to obtain a correct appreciation of the tides, gauges should be set up at several suitable points, and observations made continually at all of them, at intervals of half an hour under ordinary circumstances; but when any sounding work is going on, the height of the tide should be noted every ten minutes at the particular gauge or gauges to which the soundings that are being taken will have to be referred.

The observation is so simple that it can be made perfectly well by any man, blue-jacket or marine, of or-

dinary intelligence and steadiness, who may keep a two or four hours' watch beside the gauge, without any difficulty or hardship in good weather: in bad or severe weather the observations must be made at the times and in the manner judged best by the officer in charge of the survey; concerning which it is only necessary to note that it is important, even though it be troublesome and difficult, to make frequent observations at the changes of the moon.

When there is no strong current, a pole, heavily weighted at one end so as to float in a vertical position, may be moored with just such a length of mooring rope that it shall be straight up and down at the lowest water; the rope itself and pole attached, on which last the scale of feet is painted, thus constitutes the gauge, which may be conveniently used in the middle of the bay or harbour, or at any suitable distance, so that it can be easily observed from the ship's deck. Such a form of gauge is however not available when there is a strong current, as in the estuary of a river, where the pole would be most commonly dragged forcibly under water.

At every gauge, the time and height of high and low water each tide should be accurately observed; and it is to be especially noticed that the time of high or of low water is actually the time when the water has the greatest or least depth; and is, very frequently, *not* the time when the tidal current turns and begins to run the opposite way; is *not*, that is to say, the time of slack water. In some estuaries, in fact, the stream runs out the whole time, whether the tide is rising or falling; and in narrow

seas, in mid channel, the current does not turn for a long time after the water has ceased to rise or fall : in the English Channel, for instance, midway across, the eastward current continues to run for about three hours after high water, and the westward current for about the same time after low water. The want of a distinct understanding on this point has often caused great confusion in tidal observations ; and it must therefore be impressed on the observer that what he has principally to note are the heights and times ; the direction, times of turning and estimated rate of the current being also, though secondarily, noted in a column for remarks.

The interval in time between apparent noon and the following high water, at any place, on the days of Full or New Moon, is called the *Establishment* of that place, or more commonly the *Establishment of the Port* : it is also frequently spoken of as the *Change Tide*, and in the Nautical Almanack as well as in the Admiralty Tide Tables, is given for many places as the 'Time of High Water on Full and Change Days.' The establishment of a port, is, within a few minutes, a constant time for the port, or, more strictly, at the tide gauge ; and all calculations as to the time of high water, on particular days between Full and Change, are based on it : it should therefore be observed with great care, through as many lunations as possible.

The soundings as entered in the note book at the time, are afterwards compared, time by time, with the tidal register at the nearest gauge, or the one which, from its position, may be fairly supposed to represent the state of

the tide at the place of sounding. Whatever the register gives, at each particular time, as the height of the tide above the lowest water registered at any spring tide, is then subtracted from the depth shown by the corresponding sounding; and it is this reduced depth which is marked on the chart, usually in fathoms, but very commonly in feet when the soundings are shoal. Sometimes the reduced sounding is still further decreased by one or two feet, to allow freely for errors in the register; and if the observations extend over only a short time, so that the height of lowest tides is very imperfectly known, this additional precaution is perhaps advisable; but it is after all only a rude guess; it is an unsatisfactory way of doing work; and it would be better to give the results of the best observations, with a note as to caution; concerning which it is only necessary to add, that spring tides near the equinoxes are sometimes a foot or two higher or lower than at other seasons: and that heavy gales of wind, even at some distance from the shore, give rise to very considerable irregularities in the height of the tide. As the equinoxes seldom pass by without heavy weather, it is especially desirable to obtain tidal observations at these seasons.

CHAPTER V.

RUNNING SURVEY.

Conduct of a Running Survey—Between two known positions—Independently—Necessity of constant verification of Charts.

THE rigid accuracy of the determination of positions, distances, and angles, by the methods already laid down, cannot of course be carried out, except in places where the nature of the ship's service permits of her making a lengthened stay, or of leaving a party on shore to carry on the requisite observations: at the discretion of the officer in charge of the survey, they must be modified to suit the particular circumstances; but for a harbour, or anchorage likely to be much resorted to, or whose peculiar features in any way point it out as a convenient station of reference, or the base of an extended system of triangulation carried all along the coast, the more rigorous the manner in which the detail can be executed the better. When it is desired that the chart of a coast should be as exact as possible, it is absolutely necessary to carry on the triangulation by shore observations: but in the ordinary course of a nautical survey, a line of coast destitute of harbours or anchorages may be, and very com-

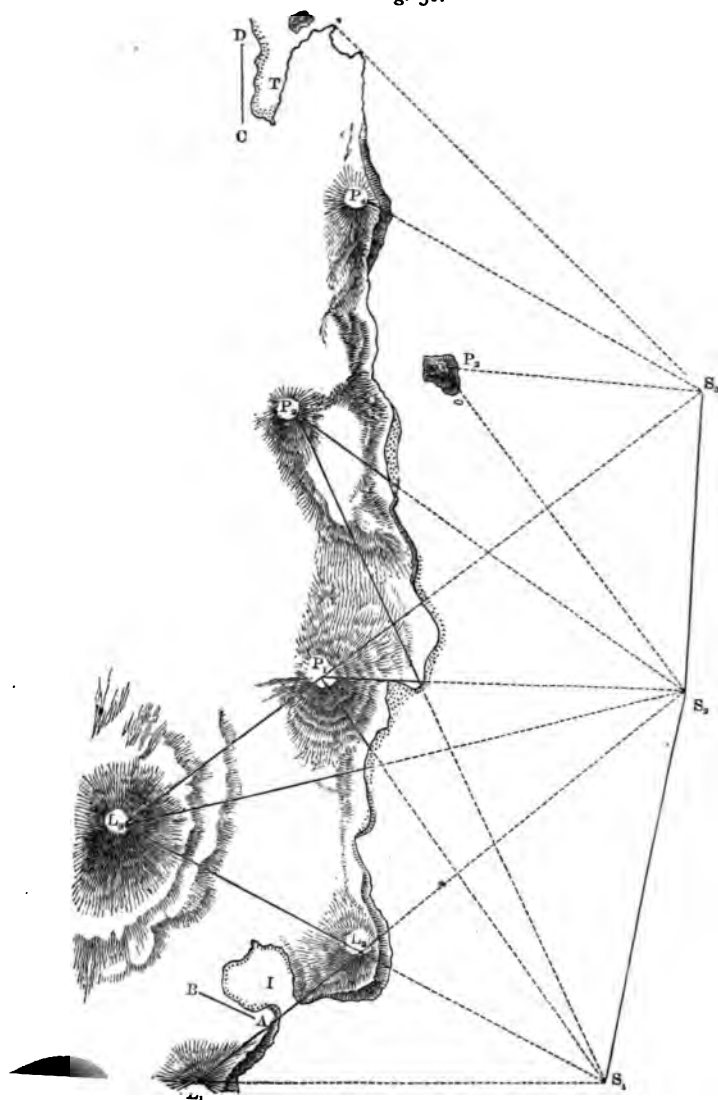
monly is charted from observations made on board ship ; the positions being checked by the final agreement with or difference from a terminal position, to which they are continued, and which is accurately determined by the most exact methods available.

Suppose for instance the harbour I , and the adjacent country, to be accurately surveyed from the initial station A , and the base line AB ; the several prominent positions $L_1, L_2, L_3 \dots$, being determined by exact observations during a stay of some days (or weeks) in the port. Leaving this port, and proceeding slowly along the coast, to T , another harbour which may be, or has been also accurately surveyed, the intervening positions may be filled in from observations of the following nature.

When the ship is in position S_1 , determined by the known positions L_2, L_3 being in line, and the angle $L_1S_1L_2$, observations are made of the angles from L_3 to the various unknown positions, $P_1, P_2, P_3 \dots$, as many as can be distinctly made out : these angles may be and should be taken both with the sextant, and the standard compass ; the one instrument is a most useful check on the other, and the observed bearings of the different positions confirm or check other and later observations.

The ship then proceeds to station S_2 , determined, as in the figure, by the positions L_2, L_1 being in line, and the angle $L_2S_2L_3$, or by such other angular observations as the necessities of the case may dictate : it is further determined by the corrected compass course and the distance by patent log ; but this last cannot be depended

Fig. 31.



on, owing to the uncertainty and irregularity of tidal currents. At this station S_2 , so determined, the angles from L_2 to the undetermined positions, $P_1, P_2 \dots$ as many as are distinctly visible, are taken, with both sextant and compass, in the same way as the similar angles were taken at the station S_1 . All the positions which have been thus observed at both the stations S_1, S_2 , are therefore determined with a certain amount of accuracy, and may be used for the successive determination of other stations from a point S_3 ; and in this manner the determination of all the prominent objects may be carried along a considerable line of coast.

These determinations are connected directly with P_n , some prominent object in the immediate neighbourhood of the second port T , from which they may be referred to a terminal station C , the position of which may be fixed by shore observations with as much accuracy as the time at disposal permits. This being done, the intermediate positions, $P_1, P_2 \dots$ which have been approximately fixed by the observations at sea may be corrected by reference to the accurate determinations of these initial and terminal stations. The difference of latitude and the meridian distance between A and C are accurately known, within a limit which can scarcely exceed 200 feet, and is probably a very great deal less; but the difference of latitude or of longitude which is accumulated by the successive determinations of the positions $P_1, P_2 \dots$, probably and indeed almost certainly will be found to differ from these very considerably. A reduction has therefore to be applied to each difference

whether of latitude or longitude, in proportion to its actual value ; and if time or weather does not permit any further observations, the positions so reduced are determined as correctly as can be under the circumstances.

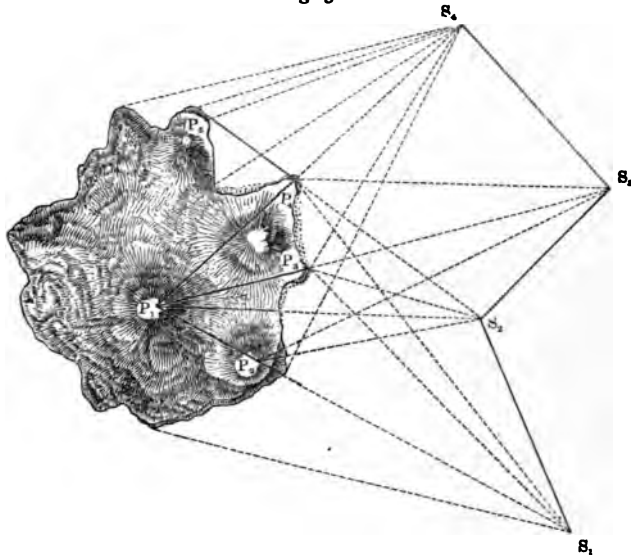
The outline of the coast is necessarily filled in at discretion ; but in a vessel specially employed in the survey, by standing into the shore as near as is consistent with safety, and by sending boats closer in, whenever it is practicable to do so, a degree of accuracy may be attained, which, considering the imperfect and rude nature of the observations, is almost surprising. Of course quick angular measurement will do a great deal ; and it is scarcely necessary to point out that angles taken to the most insignificant objects on the shore are valuable in sketching in the outline : these objects need not even be permanent ; and a lump of wood or carrion thrown up by the tide may answer all useful purposes, as well as a noticeable boulder, or a change in the nature of the beach.

It may however happen that a portion of unknown coast line has to be roughly surveyed, without landing at all, and thus without determining accurately any station of immediate reference. Such a necessity has, in fact, often arisen in the Pacific ; and even yet, a large proportion of the islands in that ocean have not been surveyed in any more exact way.

The position of the ship S_1 , is determined by astronomical observations of the usual nature, taken with every possible care, and the true bearing of some one prominent position on the shore, such as P_1 , is at the same time

found by the customary astronomical method: from this the angles are taken to every other position P_2, P_3

Fig. 32.



. . . , which can be clearly seen, both with the sextant and compass.

The second station S_2 should also be determined by astronomical observations; for the compass course and more especially the distance by patent log are not to be implicitly trusted; and in the same way, the other positions at which the ship makes observations, S_3, S_4 . . . should all be fixed so far as practicable by astronomical observations; the bearings of the principal objects at each station, should also be found in the same way, a

chain of all the possible angles being taken, which is done easily and quickly enough by a practised hand.

It is worth pointing out, that it is well, whenever possible, to see that the points $S_1, S_2 \dots$ at which the observations are taken are in line with some pair of prominent objects, whether determined or not. Thus in fig. 32, I have shown S_1 in line with P_2 and P_1 ; S_2 in line with P_4 and P_5 ; and similarly for the other stations: thus also in figure 31, where S_1 is in line with L_2 and L_3 , &c. It is an observation in which there cannot be much error; it facilitates the work of calculating or protracting; and it gives a check on the other observations which is highly useful.

The two methods, which I have just described, of obtaining a survey of a line of coast by observations from the ship, are essentially the same; the only point of difference being that in the first, the work is referred to stations accurately fixed by observations on shore, and such astronomical observations as are made from time to time are merely checks on the general series of positions; whereas in the second, the work depends entirely on the correctness of the positions fixed by observations at sea, and is therefore liable to the inaccuracies which cannot be avoided in sea observations; but with a clear sun, a good horizon, and a habit of exact observation, much may nevertheless be done.

I have already spoken of the necessity of frequently taking these observations, and of determining the positions of the ship, as $S_1, S_2 \dots$, as carefully as possible. This of course can only be done by combining the results

of the astronomical observations with the measurements made by compass and log-line, which can never be quite satisfactory. On this account, Sumner's method is perhaps the best that can be used, as it involves a smaller 'correction for run' than is necessary if the latitude has to be reduced backwards or forwards from noon to time of sights. When the sun's declination is nearly equal to the latitude, the time during which sights for longitude may be taken, is much extended; and sights before 11^h. A.M., or after 1^h. P.M., will give reliable results.

Under the same circumstances, that is, when declination and latitude are nearly the same, the longitude may be found as accurately as can be expected, by observations of equal altitudes a few minutes before and after noon. This method of finding the longitude is not, so far as I am aware, given in any of the text books of navigation, probably because its application is very limited; but when it is properly applicable, it is perfectly satisfactory. The computation is extremely simple. In the short interval, there is no appreciable change of declination; consequently no 'equation of equal altitudes:' the only work therefore, is to apply the equation of time, with its proper sign, to 0^h. 0^m. 0^s. or 24^h. 0^m. 0^s., the time of apparent noon, and to deduce the longitude in the usual way, from the Greenwich Mean Time, as determined by the chronometer, or by reference to any secondary meridian, in the manner already explained.

The equal altitudes may be observed from about 5 minutes before to 5 minutes after noon; and the mean longitude resulting from 3 or 4 pairs of such observations

may be considered quite as reliable as the longitude found by the ordinary sights, provided the sun's zenith distance is not greater than from 5° to 10° : if it is greater than that, the method is not to be trusted to as accurate; though for determining positions at sea, in the daily course of a ship's navigation, if from the state of the weather the forenoon sights have been missed, it may be used as a stand-by, better than the dead-reckoning, up to, perhaps, a meridian zenith distance of 20° ; but in this case it would be well to extend the time of taking the sights to 20 minutes before and after noon.

When almost under the sun, I have myself several times determined the longitude in this manner, by sights taken within a minute on each side of noon; but this is only possible when the zenith distance is not more than a couple of degrees, at the outside, and is a case that must evidently be quite exceptional.

In connection with these observations, it is necessary to remark that, though near the meridian, they cannot be used for finding the latitude: the second correction, and the others, which, under ordinary circumstances, are inappreciable, become considerable when the zenith distance is small. It will be seen in fact (p. 23) that they involve the cotangent of the zenith distance, $\cot m$, and its powers; so that when m is very small, expressions involving

$$x^2. \cot m, x^3. \cot^2 m, x^4. \cot^3 m \dots$$

cannot be neglected, even though x , the first reduction, is a very small quantity. This is pointed out by Raper, both in the text of his *Navigation*, and in his *Tables*.

In surveys of the nature which we have been here discussing it is frequently necessary to trust to compass observations, and it is always advisable to make them, if only as a check on the determination of positions. It must therefore be especially borne in mind that the 'Deviation' of the compass changes with the geographical position of the ship, as well as the 'Variation.' About the variation there is no difficulty, for it is now laid down with sufficient accuracy on all charts, whether in detail, or in a series of irregular curves covering the whole world : but the deviation belongs to the individual ship, and should be determined as often as opportunity offers. Every navigator is supposed to be familiar with the ordinary methods of 'swinging ship,' and any description of these would be beyond the scope of the present work : I would however call attention to this, that in surveying a strange coast without any opportunity of landing, at any considerable distance, whether in latitude or longitude, from the place where the deviation was last determined, the table of deviation then formed ought not to be trusted ; but the deviation ought again to be determined, in that particular locality, by the method used at sea ; that is, by bringing the ship's head successively to the different points of the compass, and observing the sun's azimuth, which, compared with the azimuth calculated for the same time, gives the total error of the compass, variation and deviation combined : from this, since the variation is known from the chart, the deviation is readily found. It is necessary to call attention to this here ; but the whole subject of 'Deviation' has been so

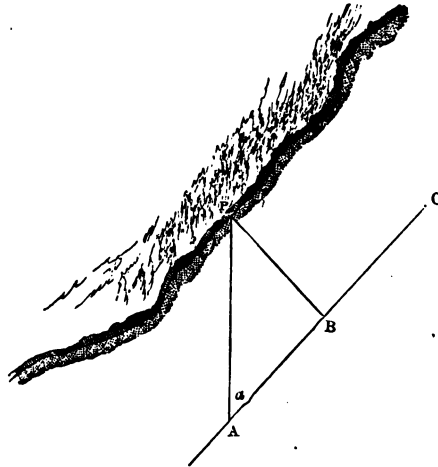
fully worked out by Staff-Captain Evans in his recent 'Elementary Manual,' that any further remarks on it would be more than superfluous. For the navigator, the compass is a most valuable instrument ; and a constant study of its peculiarities and vagaries is an essential part of his duty ; the work of the surveyor is frequently rather to point out, for the benefit of navigators, what may act as checks on the compass ; and his bearings, so far as possible, are taken independently of it. It is only when the duties of the navigator and of the surveyor coalesce, as they do in the methods of surveying which we have just been treating of, that the compass becomes indispensable ; in which case, every possible precaution to ensure its correctness must be carried out with redoubled rigour.

In filling in the outline of the coast, as many positions in it as can be, however roughly, determined by observed angles, may of course be put in pretty correctly. When such observations are numerous—and the more numerous they are, the better—the actual computation of them would be very laborious ; and the labour would be the more irksome, as the rude nature of the observations puts anything like exactness in the result out of the question. Given as a base, the distance run by ship, or boat, between two sets of observations, and also, the angles observed between the ship's (or boat's) course and the object on the beach, the distance is readily found from any table which gives a solution of these triangles : or if *one is not* on board it is no difficult matter to make one.

In doing so, the angles for every degree within the necessary limits are tabled, as arguments, along the top and down the side, the base being taken uniformly as unity : the distances corresponding to this base are inserted at the meeting of the vertical and horizontal columns belonging to the angles as arguments, and may afterwards be reduced to any required base by simple multiplication.

A very neat way of saving the trouble even of taking the distance out of the table, is by observing the angle between any object on the shore and the ship's course ;

Fig. 33.



the ship stands on, on that course, the angle being observed from time to time, and the patent log is hauled in when the angle becomes double what it was at the

first observation. The distance shown by the log is, allowing for the necessary imperfections and inaccuracies, the distance of the object from the ship at the second observation. The proof of this is easy.

Let P be a point on the shore observed from the ship at A as making with AB , the ship's course, an angle $PAB=a$. Suppose the ship to stand on to B , where the angle PBC is observed to be $2a$.

Then in triangle PAB , the exterior angle, PBC , is equal to the two interior and opposite angles,

$$\begin{aligned}\therefore \angle PBC &= \angle PAB + \angle APB, \\ \text{or } 2a &= a + \angle APB,\end{aligned}$$

$\therefore$ (by transposition)

$$\angle APB = a,$$

the triangle BAP is therefore isosceles,

$$\text{and } BP = BA;$$

that is, the distance of the point on shore is equal to the distance sailed over between the observations.

But the determinations of these very secondary positions may ordinarily be left to the protraction; and the use of any method, such as these I have pointed out, is principally as a check against slips and mistakes, which one would say were impossible, did we not know by experience that they are not only possible, but even frequent, and as such have to be constantly guarded against.

But after all, and whatever pains may be taken, a survey which does not principally depend on careful shore observations, can have no pretensions to minute accuracy.

In the early days of maritime geography, when long lines of coast were all but unknown, these methods of a running survey were very commonly practised, and were frequently, in fact, the only ones at all adapted to the circumstances. It was in some such manner that Cook and Vancouver, La Pérouse and Dentrecasteaux, Kotzebue and others, in the last and early part of the present century, surveyed the West Coast of North America, many of the Islands of the Pacific, and parts of the Asiatic and Australian coasts : from their labours, our geographical knowledge received most important additions ; our knowledge of the general outline of even the principal coasts, till then extremely vague and ill-determined, advanced with rapid strides ; and by the year 1820 or 1830, the maps of the world did not differ much in outline from those of the present time, with the exception of the Arctic and Antarctic regions, which are even still comparatively unknown. This fulness of our knowledge introduced a more rigorous and exact method of surveying : it was no longer a general and approximately correct map of a coast line which was wanted ; but an exact and minutely correct chart, drawn on a scale which admitted of such minute correctness. It is such work that the surveyors of the present day have before them ; it is such work that, in our own service, has occupied and continues to occupy the lives of men like Spratt and Shortland, and their successors ; and that, in the service of the United States, has been for years, and is still being carried on by the officers of the United States Coast Survey. In all parts of the world surveyors are at work, no longer to sketch

in a rough outline, with rude approximations, but to form a chart thoroughly trustworthy and rigidly accurate, not only above water, but below. The doing so is essentially a work of detail ; and it is on this account that I have devoted so large a proportion of these pages to a description and discussion of the most accurate methods of carrying it out. At the same time, I have endeavoured to show the degrees of accuracy which may be attained when time, weather, or other causes do not permit the utmost possible accuracy ; and it will probably be in such inferior methods that young officers will be mostly exercised. That they should be so exercised whenever opportunity offers is desirable ; for though the necessity of carrying on a detailed survey, in a ship commissioned for ordinary service, is now rare, it may occasionally happen ; and the possibility of such a necessity ought to be suitably provided against. An officer ought not to have to learn the veriest rudiments of the art, when, by the exigencies of the service, he is called on to practise it : and yet the taking of observations on shore, even with the sextant, does not come altogether by the light of nature ; whilst the choosing points for a base line or other stations, the readily picking up stations at some distance off, the numerous little artifices necessary to catch the light, even the correctly and quickly reading a sextant at night, are all things which have to be learned, and can only be learned by actual practice.

There is, however, one point of surveying which ought to be systematically carried out by every ship which lies for any time in a comparatively unfrequented harbour,

that is, a verification of the soundings. It matters nothing whether the harbour has been surveyed with rigid accuracy or not ; the soundings in, and round it, and in the approaches to it, ought to be taken and compared with the leading marks on the chart, as carefully and strictly as if they had never been taken at all.

Changes in the line of coast are not of very frequent occurrence ; and when they are met with, in all probability they at once call attention to the necessity of a new survey ; for which the old point, astronomically determined, and the original base line will naturally be used, if they have not been washed away by the inroad of the sea.

But changes in the contour of the bottom are met with every day. Harbours silt up by the tide ; if a river, or stream however small, runs into a harbour, shoals form, or shift their position ; and in a tidal estuary this shifting of the shoals is often extreme. In volcanic districts, when earthquakes of greater or less intensity are constantly going on, the bottom is often violently and suddenly altered ; and in parts of the world subject to these disturbances, no reasonable opportunity of verifying the soundings ought to be omitted.

The advisability of this proceeding has been dwelt on by successive hydrographers, in the several editions of the 'Admiralty Manual ;' and it seems indeed almost strange that it has never been formally ordered by the Admiralty ; for many instances have occurred, and within my own personal knowledge, of ships having taken

the ground in consequence of some natural change in the bottom, such as I have just spoken of.

In these days of ships with very great draught of water, able by reason of their steam power to venture closer in with the land than would have been thought prudent even a few years ago, or to try passages whose narrowness has hitherto rendered them little used, it is more than ever necessary that the soundings and the marks which fix them should be accurately laid down on the chart, and should be verified on every possible opportunity.

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